Distance Learning Initiative

Introduction to Robotics

Inverse Kinematics

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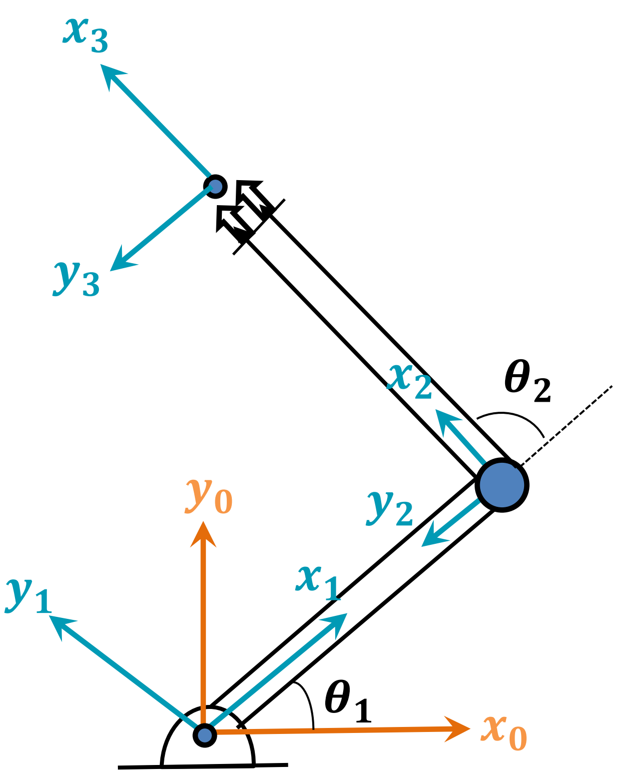
2020

Inverse Kinematics

Example:

For the planar 2DOF RR robotic arm shown in the figure, find the values of θ_1 and θ_2 required to accomplish a desired end-effector position x and y.

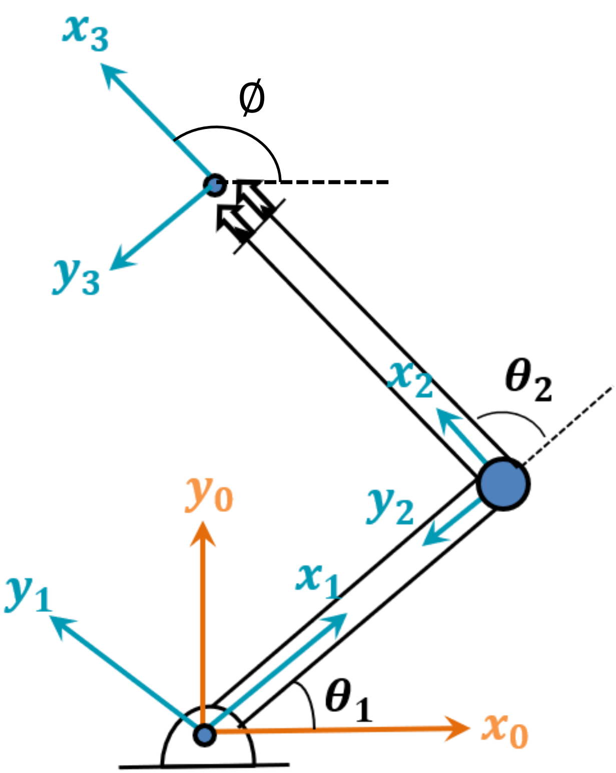
$$x = {}^0x_3$$
$$y = {}^0y_3$$



Inverse Kinematics

The transformation matrix describing the position and orientation of frame {3} with respect to frame {0} can be represented as:

$$[{}^0_3T] = \begin{bmatrix} c_\emptyset & -s_\emptyset & 0 & x \\ s_\emptyset & c_\emptyset & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

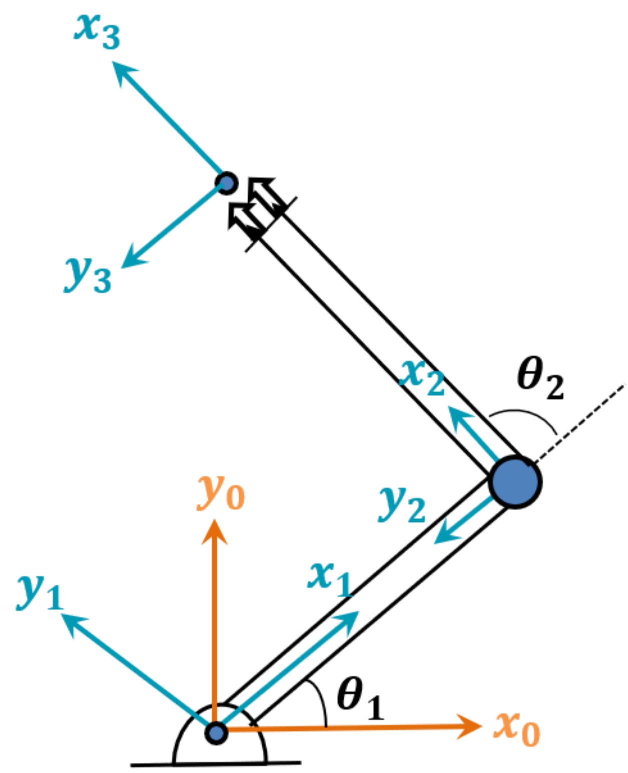


Inverse Kinematics

$$[{}^0_1T] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

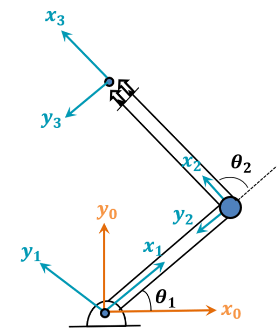
$$[{}^1_2T] = \begin{bmatrix} c_2 & -s_2 & 0 & L_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^2_3T] = \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Inverse Kinematics

$$[{}^{0}_{3}\mathrm{T}] = [{}^{0}_{1}\mathrm{T}][{}^{1}_{2}\mathrm{T}][{}^{2}_{3}\mathrm{T}]$$



$$[{}^{0}_{3}\mathrm{T}] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & L_1 \\ s_2 & c_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{0}_{3}\mathrm{T}] = \begin{bmatrix} c_1c_2 - s_1s_2 & -c_1s_2 - s_1c_2 & 0 & c_1L_1 \\ s_1c_2 + c_1s_2 & -s_1s_2 + c_1c_2 & 0 & s_1L_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{0}_{3}\mathrm{T}] = \begin{bmatrix} c_{12} & -s_{12} & 0 & c_1L_1 \\ s_{12} & c_{12} & 0 & s_1L_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Trigonometric Identities:

$\cos(a+b) = c_{ab} = c_ac_b - s_as_b$

$\cos(a-b) = c_ac_b + s_as_b$

$\sin(a+b) = s_{ab} = s_ac_b + c_as_b$

$\sin(a-b) = s_ac_b - c_as_b$

${s_a}^2 + {c_a}^2 = 1$

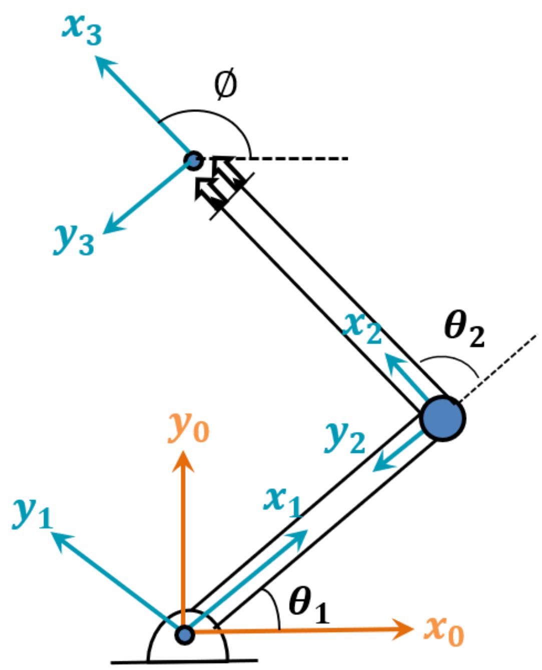
Inverse Kinematics

$$[{}^0_3\mathrm{T}] = \begin{bmatrix} c_{12} & -s_{12} & 0 & c_1 L_1 \\ s_{12} & c_{12} & 0 & s_1 L_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & L_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^0_3\mathrm{T}] = \begin{bmatrix} c_{12} & -s_{12} & 0 & c_{12} L_2 + c_1 L_1 \\ s_{12} & c_{12} & 0 & s_{12} L_2 + s_1 L_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Compare this to

$$[{}^0_3\mathrm{T}] = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_\emptyset & -s_\emptyset & 0 & x \\ s_\emptyset & c_\emptyset & 0 & y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$c_\emptyset = c_{12}$

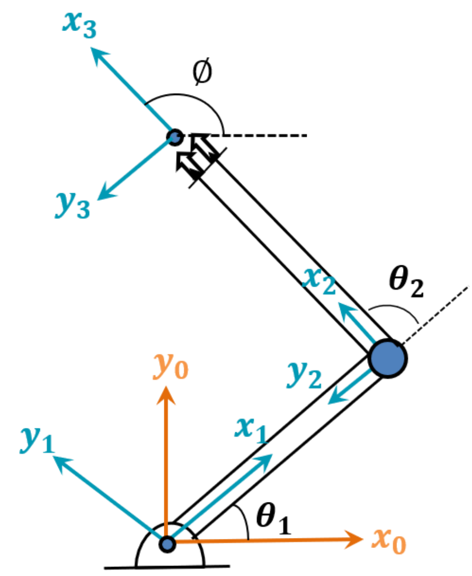
$s_\emptyset = s_{12}$

$x = c_{12} L_2 + c_1 L_1$

$y = s_{12} L_2 + s_1 L_1$

Inverse Kinematics

$c_\emptyset = c_{12}$ ------------------------- (1)
$s_\emptyset = s_{12}$ ------------------------- (2)
$x = c_{12}L_2 + c_1L_1$ --------------- (3)
$y = s_{12}L_2 + s_1L_1$ --------------- (4)



$[\text{Eqn}(3)]^2 + [\text{Eqn}(4)]^2$:

$$x^2 + y^2 = (c_{12}L_2 + c_1L_1)^2 + (s_{12}L_2 + s_1L_1)^2$$

$$x^2 + y^2 = {c_{12}}^2{L_2}^2 + 2c_{12}L_2c_1L_1 + {c_1}^2{L_1}^2 + {s_{12}}^2{L_2}^2 + 2s_{12}L_2s_1L_1 + {s_1}^2{L_1}^2$$

$$x^2 + y^2 = ({c_{12}}^2 + {s_{12}}^2){L_2}^2 + ({c_1}^2 + {s_1}^2){L_1}^2 + 2c_{12}L_2c_1L_1 + 2s_{12}L_2s_1L_1$$

$$x^2 + y^2 = {L_2}^2 + {L_1}^2 + 2(c_{12}c_1 + s_{12}s_1)L_2L_1$$

$$x^2 + y^2 = {L_1}^2 + {L_2}^2 + 2L_1L_2c_2$$

Trigonometric Identities:
$\cos(a+b) = c_{ab} = c_ac_b - s_as_b$
$\cos(a-b) = c_ac_b + s_as_b$
$\sin(a+b) = s_{ab} = s_ac_b + c_as_b$
$\sin(a-b) = s_ac_b - c_as_b$
${s_a}^2 + {c_a}^2 = 1$

Inverse Kinematics

$$x^2 + y^2 = {L_1}^2 + {L_2}^2 + 2L_1L_2c_2$$

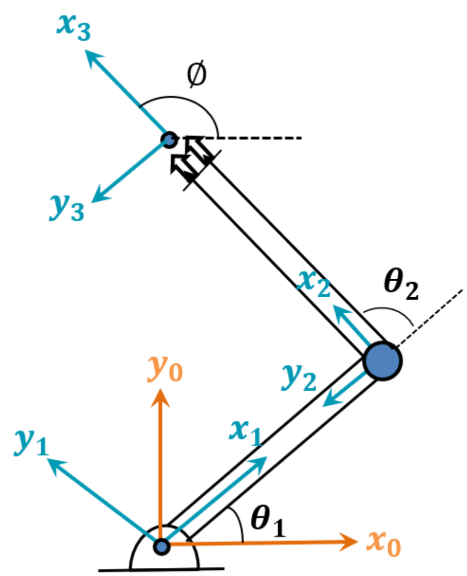
$$c_2 = \frac{x^2 + y^2 - {L_1}^2 - {L_2}^2}{2L_1L_2}$$

$$c_2^2 + s_2^2 = 1$$

$s_2 = \pm\sqrt{1 - c_2^2}$; → two solutions

$$\tan(\theta_2) = \frac{s_2}{c_2}$$

$$\theta_2 = Atan2\left(\frac{s_2}{c_2}\right)$$



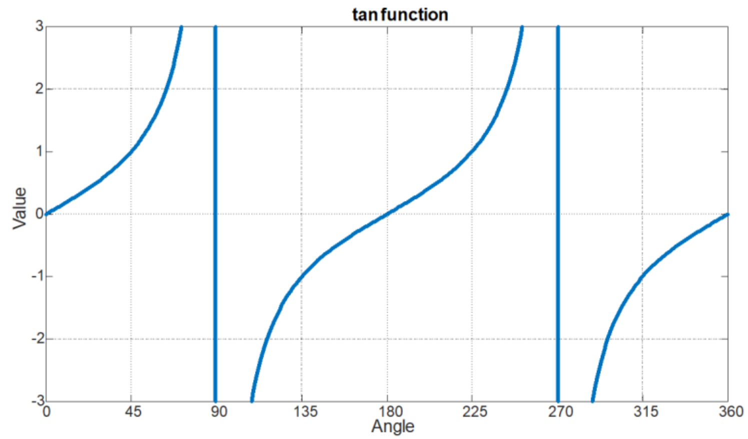
$c_\emptyset = c_{12}$ --------------- (1)
$s_\emptyset = s_{12}$ --------------- (2)
$x = c_{12}L_2 + c_1L_1$ ----- (3)
$y = s_{12}L_2 + s_1L_1$ ----- (4)

Inverse Kinematics

Command Window

```
>> help atan
 atan   Inverse tangent, result in radians.
    atan(X) is the arctangent of the elements of X.

    See also atan2, tan, atand, atan2d.

    Reference page for atan
    Other functions named atan

>> help atand
 atand  Inverse tangent, result in degrees.
    atand(X) is the inverse tangent, expressed in degrees,
    of the elements of X.

    Class support for input X:
       float: double, single

    See also tand, atan2d, atan.

    Reference page for atand
    Other functions named atand

>> help atan2d
 atan2d  Four quadrant inverse tangent, result in degrees.
    atan2d(Y,X) is the four quadrant arctangent of the elements of X and Y
    such that -180 <= atan2d(Y,X) <= 180. X and Y must have compatible
    sizes. In the simplest cases, they can be the same size or one can be a
    scalar. Two inputs have compatible sizes if, for every dimension, the
    dimension sizes of the inputs are either the same or one of them is 1.

    See also atand, atan2.

    Reference page for atan2d
    Other functions named atan2d
```

```
x = linspace(0,360,360);
y = tand(x);
plot (x,y,'Linewidth',6)
xlim([0 360])
ylim([-3 3])
xticks([0 45 90 135 180 225 270 315 360])
title('tan function', 'FontSize', 18)
xlabel('Angle', 'FontSize', 18)
ylabel('Value', 'FontSize', 18)
set(gca,'FontSize',18)
grid on
```



Inverse Kinematics

$$c_2 = \frac{x^2+y^2-L_1{}^2-L_2{}^2}{2L_1L_2};\ s_2 = \pm\sqrt{1-c_2{}^2}\ ;\ \theta_2 = Atan2\left(\frac{s_2}{c_2}\right)$$

Now, after finding θ_2, to find θ_1

From Eqn (3): $x = c_{12}L_2 + c_1L_1$

$x = (c_1c_2 - s_1s_2)L_2 + c_1L_1$

$x = c_1c_2L_2 - s_1s_2L_2 + c_1L_1$

$x = (c_2L_2 + L_1)c_1 - (s_2L_2)s_1$

Let $K_1 = c_2L_2 + L_1$ and $K_2 = s_2L_2$

$x = K_1c_1 - K_2s_1$

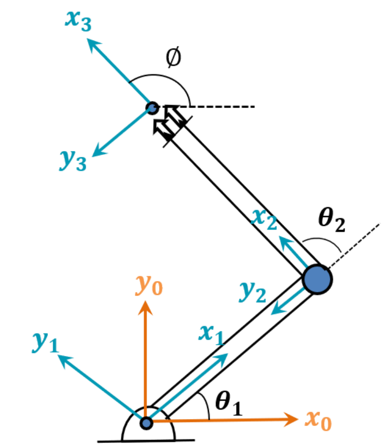
From Eqn (4): $y = s_{12}L_2 + s_1L_1$

$x = (s_1c_2 + c_1s_2)L_2 + s_1L_1$

$x = s_1c_2L_2 + c_1s_2L_2 + s_1L_1$

$x = (s_2L_2)c_1 + (c_2L_2 + L_1)s_1$

$y = K_2c_1 + K_1s_1$



$c_\emptyset = c_{12}$ --------------- (1)

$s_\emptyset = s_{12}$ --------------- (2)

$x = c_{12}L_2 + c_1L_1$ ----- (3)

$y = s_{12}L_2 + s_1L_1$ ----- (4)

Trigonometric Identities:

$\cos(a+b) = c_{ab} = c_ac_b - s_as_b$

$\cos(a-b) = c_ac_b + s_as_b$

$\sin(a+b) = s_{ab} = s_ac_b + c_as_b$

$\sin(a-b) = s_ac_b - c_as_b$

$s_a{}^2 + c_a{}^2 = 1$

Inverse Kinematics

$c_2 = \frac{x^2+y^2-{L_1}^2-{L_2}^2}{2L_1L_2}$; $s_2 = \pm\sqrt{1-{c_2}^2}$; $\theta_2 = Atan2\left(\frac{s_2}{c_2}\right)$

$K_1 = c_2L_2 + L_1$ and $K_2 = s_2L_2$

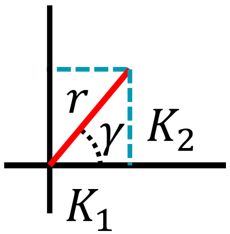
$x = K_1c_1 - K_2s_1$

$y = K_2c_1 + K_1s_1$

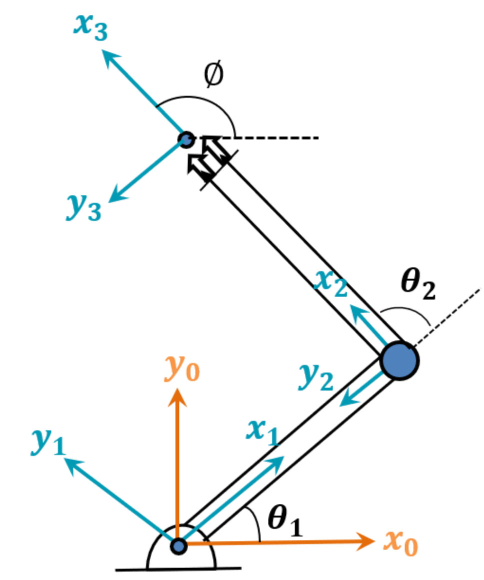
To solve these two non-algebraic equations,

Let $r = \sqrt{(K_1)^2 + (K_2)^2}$ and

$\gamma = Atan2\left(\frac{K_2}{K_1}\right)$



Then: $K_1 = rc_\gamma$; $K_2 = rs_\gamma$



$c_\emptyset = c_{12}$ --------------- (1)

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$x = c_{12}L_2 + c_1L_1$ ----- (3)

$y = s_{12}L_2 + s_1L_1$ ----- (4)

Inverse Kinematics

$$c_2 = \frac{x^2+y^2-{L_1}^2-{L_2}^2}{2L_1L_2};\ s_2 = \pm\sqrt{1-{c_2}^2}\ ;\ \theta_2 = Atan2\left(\frac{s_2}{c_2}\right)$$

$K_1 = c_2L_2 + L_1$ and $K_2 = s_2L_2$

$$r = \sqrt{(K_1)^2 + (K_2)^2};\ \gamma = Atan2\left(\frac{K_2}{K_1}\right)$$

Substitute: $K_1 = rc_\gamma$ and $K_2 = rs_\gamma$ in

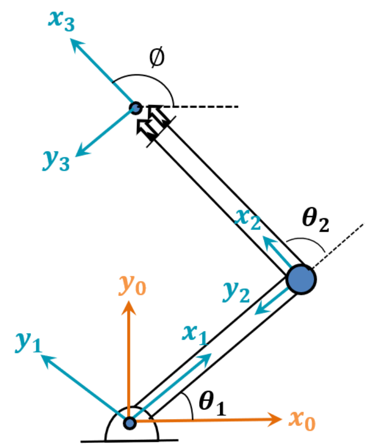
$x = K_1c_1 - K_2s_1$ and $y = K_2c_1 + K_1s_1$

To give

$$x = rc_\gamma c_1 - rs_\gamma s_1 \rightarrow \frac{x}{r} = \cos(\theta_1 + \gamma)$$

$$y = rs_\gamma c_1 + rc_\gamma s_1 \rightarrow \frac{y}{r} = \sin(\theta_1 + \gamma)$$

$$\tan(\theta_1 + \gamma) = \frac{y}{x}$$



$c_\emptyset = c_{12}$ --------------- (1)

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$x = c_{12}L_2 + c_1L_1$ ----- (3)

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$\cos(a-b) = c_ac_b + s_as_b$

$\sin(a+b) = s_{ab} = s_ac_b + c_as_b$

$\sin(a-b) = s_ac_b - c_as_b$

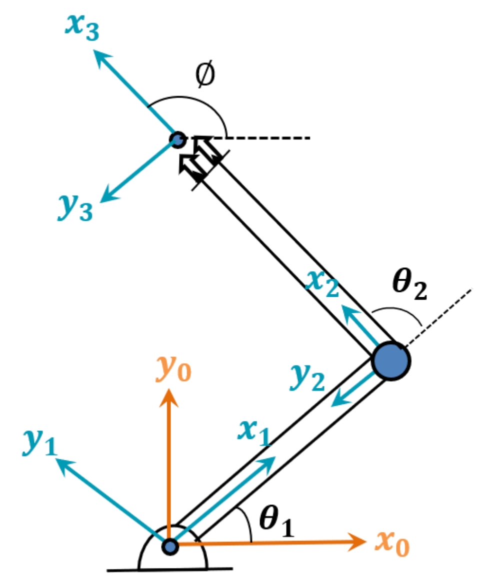
${s_a}^2 + {c_a}^2 = 1$

Inverse Kinematics

$$c_2 = \frac{x^2+y^2-{L_1}^2-{L_2}^2}{2L_1L_2};\ s_2 = \pm\sqrt{1-{c_2}^2}\ ;\ \theta_2 = Atan2\left(\frac{s_2}{c_2}\right)$$

$K_1 = c_2L_2 + L_1$ and $K_2 = s_2L_2$

$$r = \sqrt{(K_1)^2 + (K_2)^2};\quad \gamma = Atan2\left(\frac{K_2}{K_1}\right)$$



$$\tan(\theta_1+\gamma) = \frac{y}{x}$$

$$\theta_1 + \gamma = Atan2\left(\frac{y}{x}\right)$$

$$\theta_1 = Atan2\left(\frac{y}{x}\right) - \gamma$$

$$\theta_1 = Atan2\left(\frac{y}{x}\right) - Atan2\left(\frac{K_2}{K_1}\right)$$

$$\theta_1 = Atan2\left(\frac{y}{x}\right) - Atan2\left(\frac{s_2L_2}{c_2L_2+L_1}\right)$$

$c_\emptyset = c_{12}$ --------------- (1)
$s_\emptyset = s_{12}$ --------------- (2)
$x = c_{12}L_2 + c_1L_1$ ----- (3)
$y = s_{12}L_2 + s_1L_1$ ----- (4)

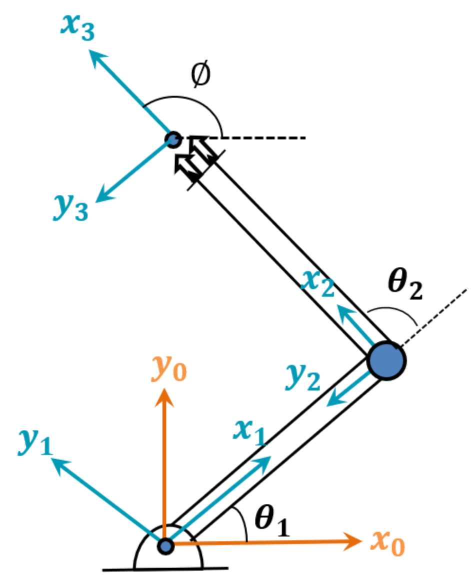
Inverse Kinematics

$$c_2 = \frac{x^2+y^2-{L_1}^2-{L_2}^2}{2L_1L_2};$$

$$s_2 = \pm\sqrt{1-{c_2}^2}\ ;$$

$$\theta_2 = Atan2\left(\frac{s_2}{c_2}\right)$$

$$\theta_1 = Atan2\left(\frac{y}{x}\right) - Atan2\left(\frac{s_2L_2}{c_2L_2+L_1}\right)$$



x & y → c_2 → s_2 → θ_2 → θ_1

x & y → c_2 → s_2 → θ_2 → θ_1

$c_2 = \frac{x^2+y^2-{L_1}^2-{L_2}^2}{2L_1L_2}$ | $s_2 = \pm\sqrt{1-{c_2}^2}$ | $\theta_2 = Atan2\left(\frac{s_2}{c_2}\right)$ | $Atan2\left(\frac{y}{x}\right) - Atan2\left(\frac{s_2L_2}{c_2L_2+L_1}\right)$

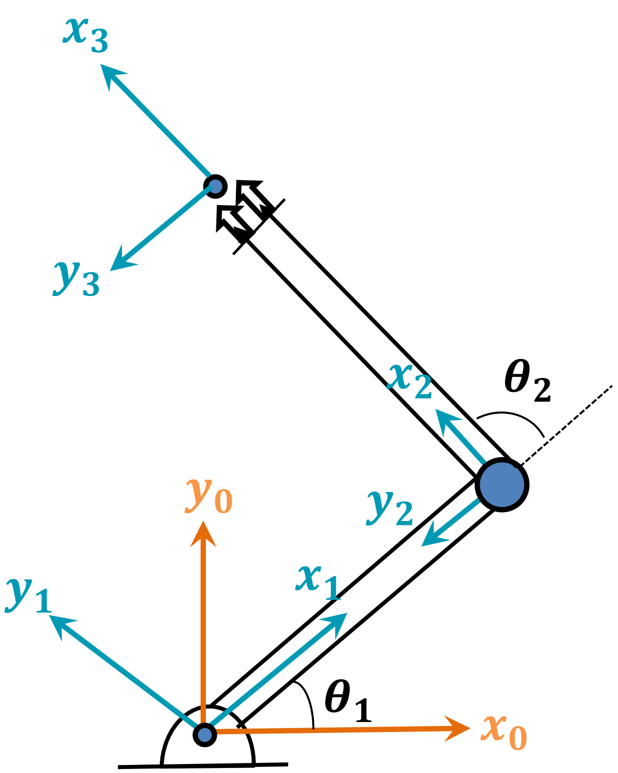
Inverse Kinematics

Example:

For the planar 2DOF RR robotic arm shown in the figure, find the values of θ_1 and θ_2 required to accomplish a desired end-effector position
$x = -1.12\ \text{cm}$ and $y = 24.52\ \text{cm}.$

Note that:

$L_1 = 25\ \text{cm}$ and $L_2 = 20\ \text{cm}$



Inverse Kinematics

$$c_2 = \frac{x^2 + y^2 - {L_1}^2 - {L_2}^2}{2L_1L_2} = \frac{(-1.12)^2 + (24.52)^2 - (25)^2 - (20)^2}{2(25)(20)} = -0.4225$$

$$s_2 = \pm\sqrt{1 - {c_2}^2} = \pm\sqrt{1 - (-0.4225\,)^2} = \pm 0.9064$$

```
>> atan2d (+0.9064,-0.4225)

ans =

  114.9916

>> atan2d (-0.9064,-0.4225)

ans =

 -114.9916
```

$$\theta_2 = Atan2\left(\frac{s_2}{c_2}\right)$$

$\theta_2 = Atan2\left(\frac{+0.9064}{-0.4225}\right)$ or $\theta_2 = Atan2\left(\frac{-0.9064}{-0.4225}\right)$

$\theta_2 = 115^o$ or $\theta_2 = 245^o \quad (= -115^o + 360^o\,)$

Inverse Kinematics

$$c_2 = \frac{x^2 + y^2 - {L_1}^2 - {L_2}^2}{2L_1L_2} = \frac{(-1.12)^2 + (24.52)^2 - (25)^2 - (20)^2}{2(25)(20)} = -0.4225$$

Note: $\theta_2 = \text{Acos}(-0.4225\ ) = 115^o$

$$s_2 = \pm\sqrt{1 - {c_2}^2} = \pm\sqrt{1 - (-0.4225\ )^2} = \pm 0.9064$$

Note: $\theta_2 = \text{Asin}(0.9064\ ) = 65^o$ or $\theta_2 = \text{Asin}(-0.9064\ ) = -65^o$

Note: $\theta_2 = Atan\left(\frac{s_2}{c_2}\right)$

$\theta_2 = Atan\left(\frac{+0.9064}{-0.4225}\right)$ or $\theta_2 = Atan\left(\frac{-0.9064}{-0.4225}\right)$

$\theta_2 = -65^o$ or $\theta_2 = 65^o$

sin	All
tan	cos

$$\theta_2 = Atan2\left(\frac{s_2}{c_2}\right)$$

$\theta_2 = Atan2\left(\frac{+0.9064}{-0.4225}\right)$ or $\theta_2 = Atan2\left(\frac{-0.9064}{-0.4225}\right)$

$\theta_2 = 115^o \quad (= -65^o + 180^o)$ or $\theta_2 = 245^o \quad (= 65^o + 180^o)$

Inverse Kinematics

$$\theta_1 = Atan2\left(\frac{y}{x}\right) - Atan2\left(\frac{s_2 L_2}{c_2 L_2 + L_1}\right)$$

$\theta_2 = 115^o$:

$$\theta_1 = Atan2\left(\frac{24.52}{-1.12}\right) - Atan2\left(\frac{20\sin(115)}{20\cos 115 + 25}\right) = 45^o$$

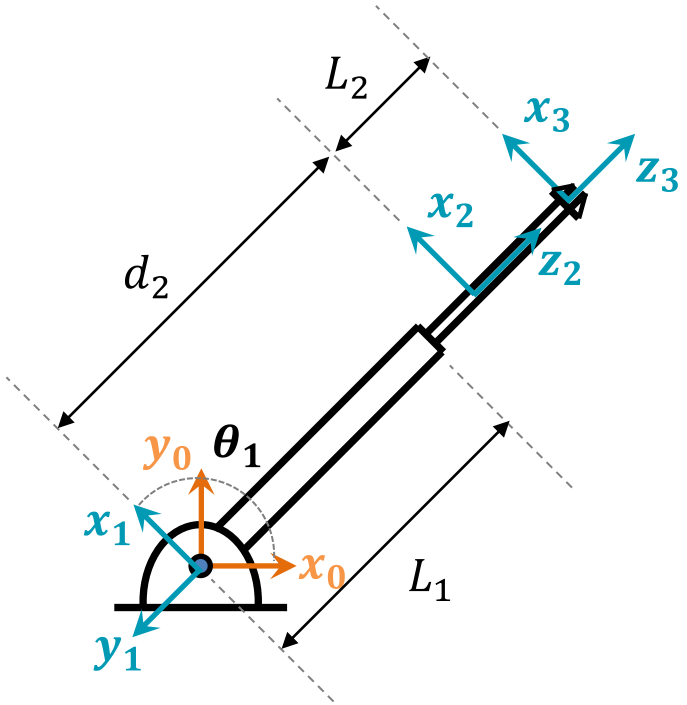
or

$\theta_2 = 245^o$

$$\theta_1 = Atan2\left(\frac{24.52}{-1.12}\right) - Atan2\left(\frac{20\sin(245)}{20\cos 245 + 25}\right) = 140^o$$

Example

Example: Perform the inverse kinematics analysis for the planar 2 DOF RP robotic arm show in the figure?



Forward Kinematics

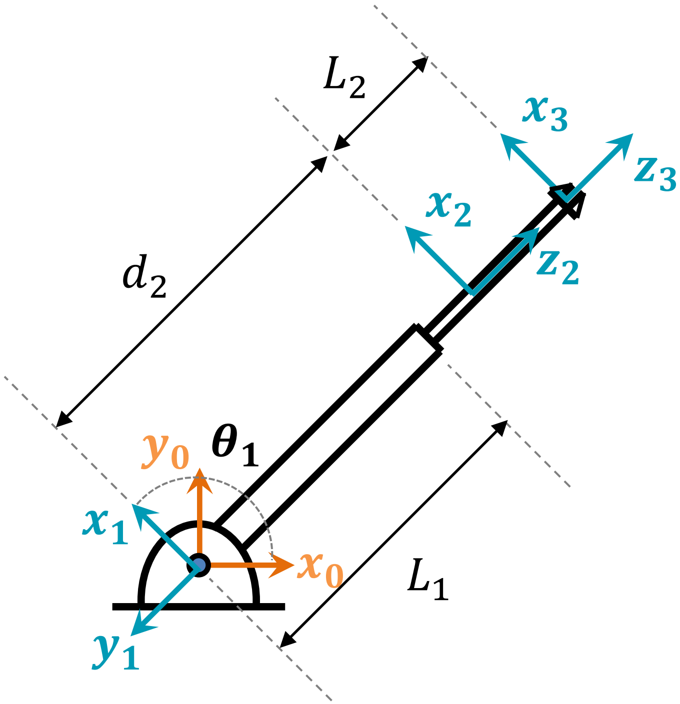
Forward Kinematics:

$$[{}^{i-1}_{i}T] = \begin{bmatrix} c_{\theta i} & -s_{\theta i} & 0 & a_{i-1} \\ s_{\theta i}c_{\alpha i-1} & c_{\theta i}c_{\alpha i-1} & -s_{\alpha i-1} & -d_i s_{\alpha i-1} \\ s_{\theta i}s_{\alpha i-1} & c_{\theta i}s_{\alpha i-1} & c_{\alpha i-1} & d_i c_{\alpha i-1} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{0}_{1}T] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{1}_{2}T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{2}_{3}T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



i	a_{i-1}	α_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	90	d_2	0
3	0	0	L_2	0

Forward Kinematics

$$[{}^0_3\mathrm{T}] = [{}^0_1\mathrm{T}][{}^1_2\mathrm{T}][{}^2_3\mathrm{T}]$$

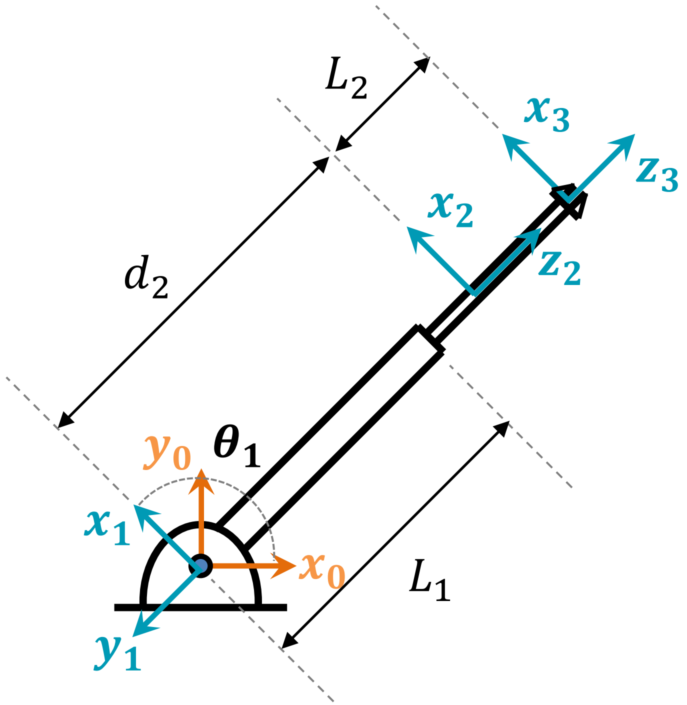
$$[{}^0_3\mathrm{T}] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^0_3\mathrm{T}] = \begin{bmatrix} c_1 & 0 & s_1 & d_2 s_1 \\ s_1 & 0 & -c_1 & -d_2 c_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^0_3\mathrm{T}] = \begin{bmatrix} c_1 & 0 & s_1 & L_2 s_1 + d_2 s_1 \\ s_1 & 0 & -c_1 & -L_2 c_1 - d_2 c_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c_1 & 0 & s_1 & (L_2 + d_2) s_1 \\ s_1 & 0 & -c_1 & -(L_2 + d_2) c_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Forward Kinematics:

$$[{}^0_3\mathrm{T}] = \begin{bmatrix} c_1 & 0 & s_1 & (L_2 + d_2)s_1 \\ s_1 & 0 & -c_1 & -(L_2 + d_2)c_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Inverse Kinematics

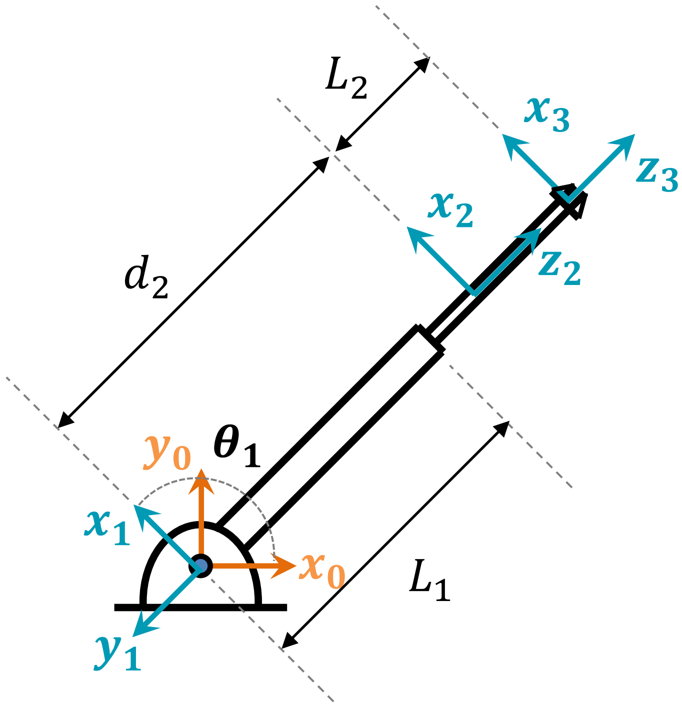
Inverse Kinematics:

$$[{}^{0}_{3}\mathrm{T}] = \begin{bmatrix} c_1 & 0 & s_1 & (L_2 + d_2)s_1 \\ s_1 & 0 & -c_1 & -(L_2 + d_2)c_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Compare with:

$$[{}^{0}_{3}\mathrm{T}] = \begin{bmatrix} r_{11} & r_{12} & r_{13} & x \\ r_{21} & r_{22} & r_{23} & y \\ r_{31} & r_{32} & r_{33} & z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = (L_2 + d_2)s_1$$
$$y = -(L_2 + d_2)c_1$$



Inverse Kinematics

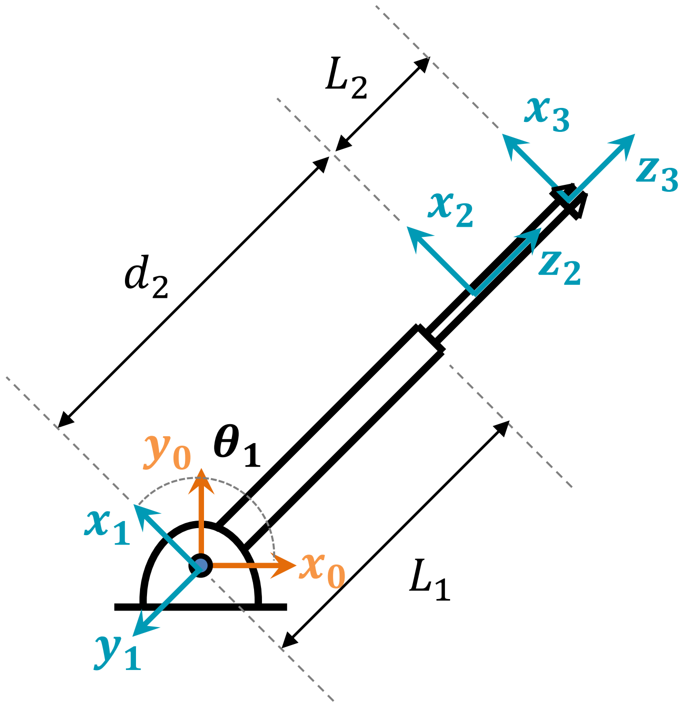
$$x = (L_2 + d_2)s_1 \qquad (1)$$
$$y = -(L_2 + d_2)c_1 \qquad (2)$$

Divide Eqn (1) by Eqn (2):

$$\frac{x}{-y} = \frac{(L_2 + d_2)s_1}{(L_2 + d_2)c_1}$$

$$\frac{x}{-y} = \tan(\theta_1)$$

$$\theta_1 = \text{Atan2}\left(\frac{x}{-y}\right)$$

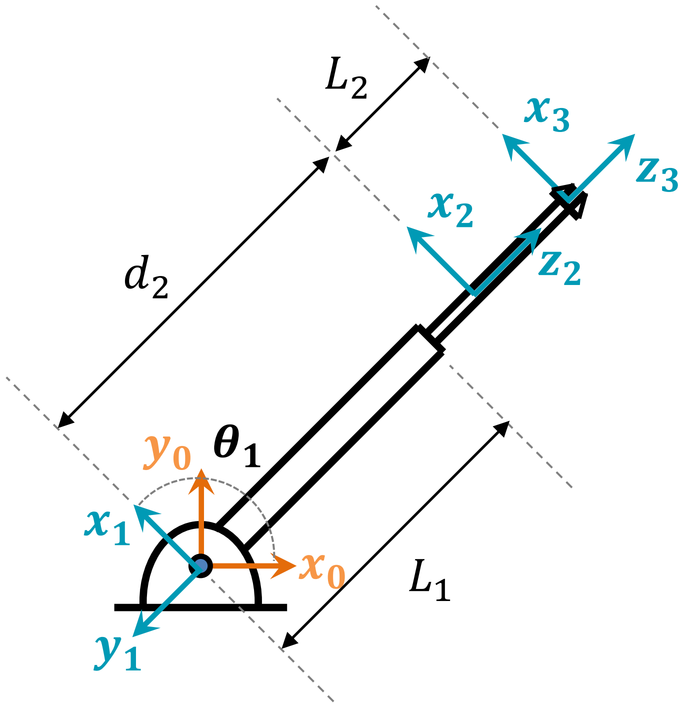


Inverse Kinematics

$$x = (L_2 + d_2)s_1 \quad (1)$$

$$y = -(L_2 + d_2)c_1 \quad (2)$$

$$\theta_1 = \text{Atan2}\left(\frac{x}{-y}\right)$$



$$x = L_2 s_1 + d_2 s_1$$

$$d_2 s_1 = x - L_2 s_1$$

$$d_2 = \frac{x - L_2 s_1}{s_1}$$

Inverse Kinematics

For more details on this subject, please see:

- Introduction to Robotics: Mechanics and Control, by John J. Craig, 3rd Edition, Addison-Wesley Publishing Company, 2003.

- M. Farman, M. Al-Shaibah, Z. Aoraiath, and F. Jarrar, “Design of a three degrees of freedom robotic arm,” International Journal of Computer Applications, vol. 179(37), pp. 12-17, 2018.