Distance Learning Initiative

Introduction to Robotics

Singularities

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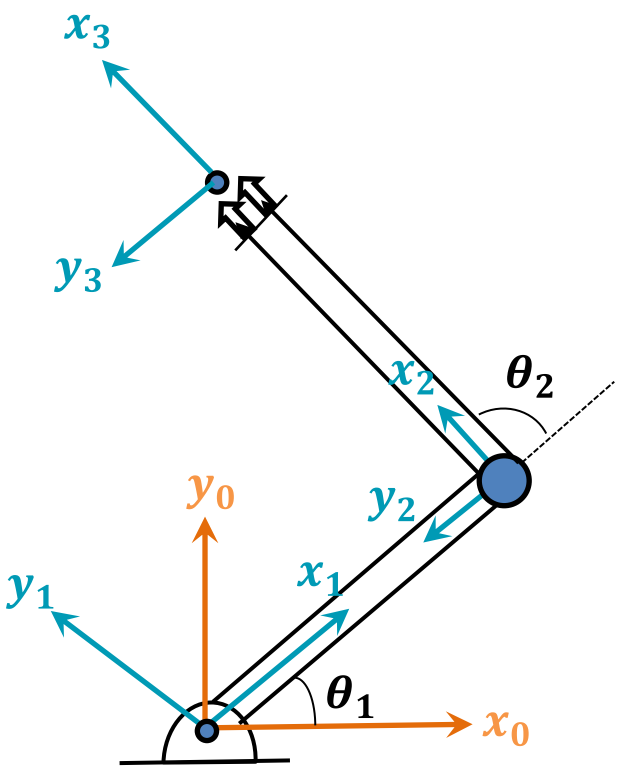
Singularities

Example:

For a planar 2 DOF RR robotic arm

$$^{3}\begin{bmatrix}\dot{x}_3 \\ \dot{y}_3\end{bmatrix} = \begin{bmatrix} L_1 s_2 & 0 \\ L_1 c_2 + L_2 & L_2 \end{bmatrix} \begin{bmatrix}\dot{\theta}_1 \\ \dot{\theta}_2\end{bmatrix}$$

Under which condition, would there be a singularity for this manipulator?



Singularities

$$^{3}\begin{bmatrix}\dot{x}_3\\ \dot{y}_3\end{bmatrix} = \begin{bmatrix}L_1 s_2 & 0\\ L_1 c_2 + L_2 & L_2\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$$

$$\begin{bmatrix}L_1 s_2 & 0\\ L_1 c_2 + L_2 & l_2\end{bmatrix}^{-1}\ ^{3}\begin{bmatrix}\dot{x}_3\\ \dot{y}_3\end{bmatrix} = \begin{bmatrix}L_1 s_2 & 0\\ L_1 c_2 + L_2 & L_2\end{bmatrix}^{-1}\begin{bmatrix}L_1 s_2 & 0\\ L_1 c_2 + L_2 & L_2\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$$

$$\begin{bmatrix}L_1 s_2 & 0\\ L_1 c_2 + L_2 & L_2\end{bmatrix}^{-1}\ ^{3}\begin{bmatrix}\dot{x}_3\\ \dot{y}_3\end{bmatrix} = \begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$$

$$\begin{bmatrix}L_1 s_2 & 0\\ L_1 c_2 + L_2 & L_2\end{bmatrix}^{-1}\ ^{3}\begin{bmatrix}\dot{x}_3\\ \dot{y}_3\end{bmatrix} = \begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix}$$

$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} = \begin{bmatrix}L_1 s_2 & 0\\ L_1 c_2 + L_2 & L_2\end{bmatrix}^{-1}\ ^{3}\begin{bmatrix}\dot{x}_3\\ \dot{y}_3\end{bmatrix}$$

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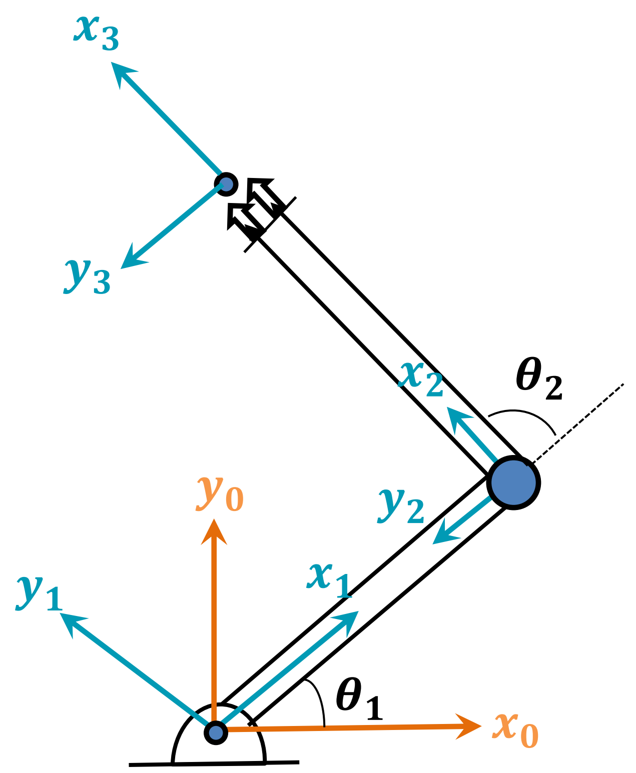
$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \begin{bmatrix} L_1 s_2 & 0 \\ L_1 c_2 + L_2 & L_2 \end{bmatrix}^{-1} {}^3\begin{bmatrix} \dot{x}_3 \\ \dot{y}_3 \end{bmatrix}$$

$$\begin{bmatrix} L_1 s_2 & 0 \\ L_1 c_2 + L_2 & L_2 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} L_2 & 0 \\ -(L_1 c_2 + L_2) & L_1 s_2 \end{bmatrix}}{L_1 L_2 s_2}$$

$$\begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} = \frac{1}{L_1 L_2 s_2} \begin{bmatrix} L_2 & 0 \\ -(L_1 c_2 + L_2) & L_1 s_2 \end{bmatrix} {}^3\begin{bmatrix} \dot{x}_3 \\ \dot{y}_3 \end{bmatrix}$$



- When would the determinant of the Jacobian matrix become equal to zero?
- What does it mathematically mean to have the determinant of the Jacobian matrix equals zero?
- What does it physically mean to have the determinant of the Jacobian matrix equals zero?

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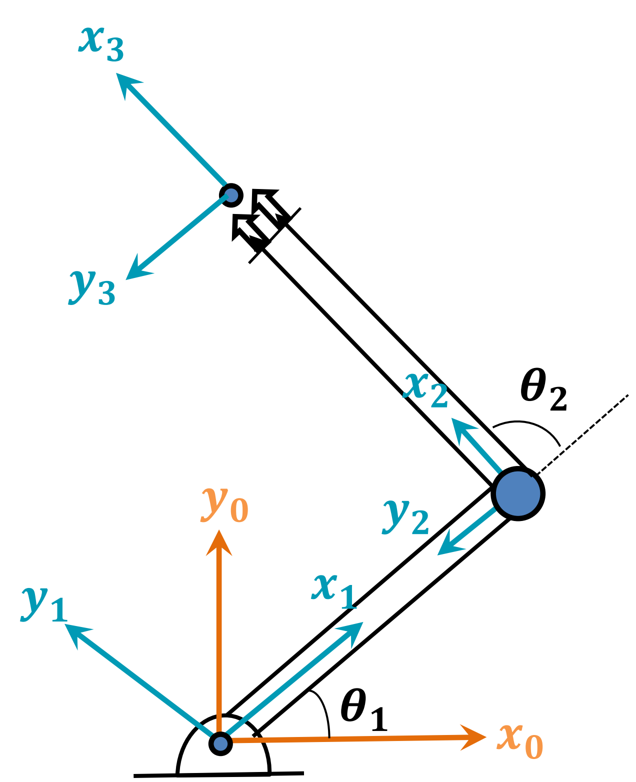
$$\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\end{bmatrix} = \frac{1}{L_1 L_2 s_2}\begin{bmatrix} L_2 & 0 \\ -(L_1 c_2 + L_2) & L_1 s_2 \end{bmatrix} {}^3\begin{bmatrix}\dot{x}_3\\ \dot{y}_3\end{bmatrix}$$

When $\theta_2 = 0^o$ (stretched completely out), or when $\theta_2 = 180^o$ (folded back), the Jacobian will be a singular matrix and thus has no inverse.

This means that a solution for the above equation would not exist.

It would require the joint rates to reach extremely large magnitudes going up to infinity, in order to acquire certain E. F. linear velocities.



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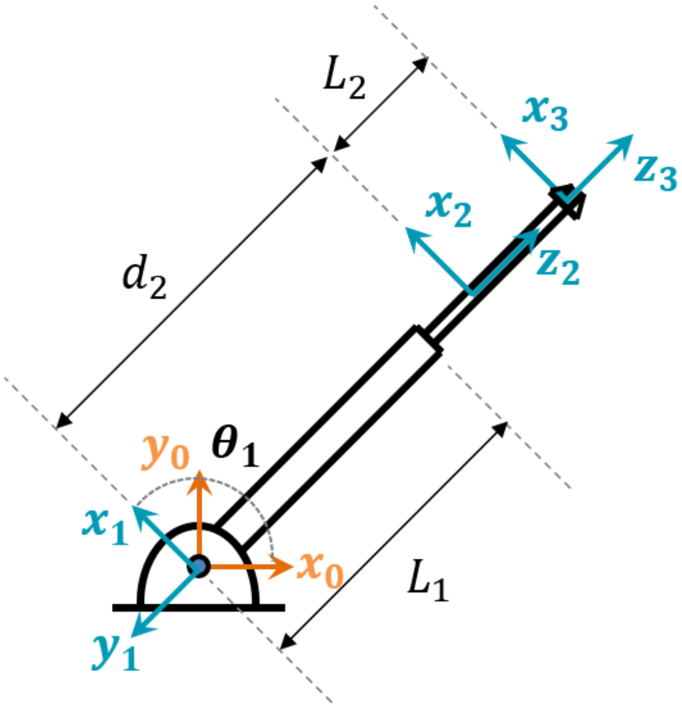
Example:

For a planar 2 DOF RP robotic arm

$$^{0}\begin{bmatrix}\dot{x}_3 \\ \dot{y}_3\end{bmatrix} = \begin{bmatrix}c_1(d_2+L_2) & s_1 \\ s_1(d_2+L_2) & -c_1\end{bmatrix}\begin{bmatrix}\dot{\theta}_1 \\ \dot{d}_2\end{bmatrix}$$

Under which condition, would there be a singularity for this manipulator?



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$$^{0}\begin{bmatrix}\dot{x}_3\\ \dot{y}_3\end{bmatrix} = \begin{bmatrix}c_1(d_2+L_2) & s_1\\ s_1(d_2+L_2) & -c_1\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\\ \dot{d}_2\end{bmatrix}$$

$$^{0}[\mathrm{J}_v]_{2\times 2} = \begin{bmatrix}c_1(d_2+L_2) & s_1\\ s_1(d_2+L_2) & -c_1\end{bmatrix}$$

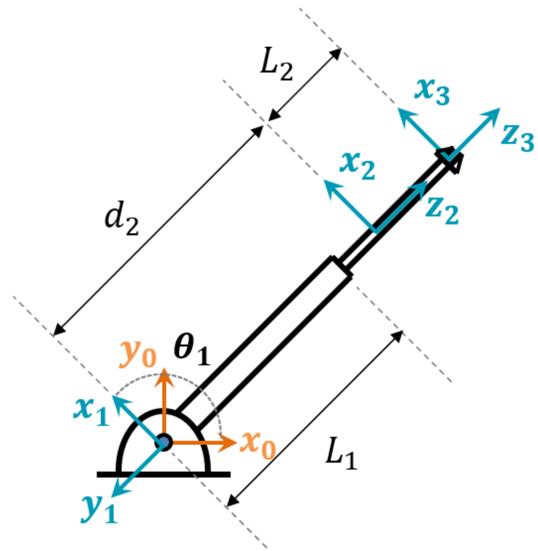
$$|\mathrm{J}_v| = c_1(d_2+L_2)(-c_1) - s_1 s_1(d_2+L_2)$$

$$|\mathrm{J}_v| = -(d_2+L_2)(c_1)^2 - (s_1)^2(d_2+L_2)$$

Set $|\mathrm{J}_v| = 0$

$$(d_2+L_2)\left((c_1)^2+(s_1)^2\right) = 0$$

$$d_2 = -L_2$$



Singularities

For more details on this subject, please see:

- Introduction to Robotics: Mechanics and Control, by John J. Craig, 3rd Edition, Addison-Wesley Publishing Company, 2003.