Distance Learning Initiative

Introduction to Robotics

Robotic Arm Link Velocities

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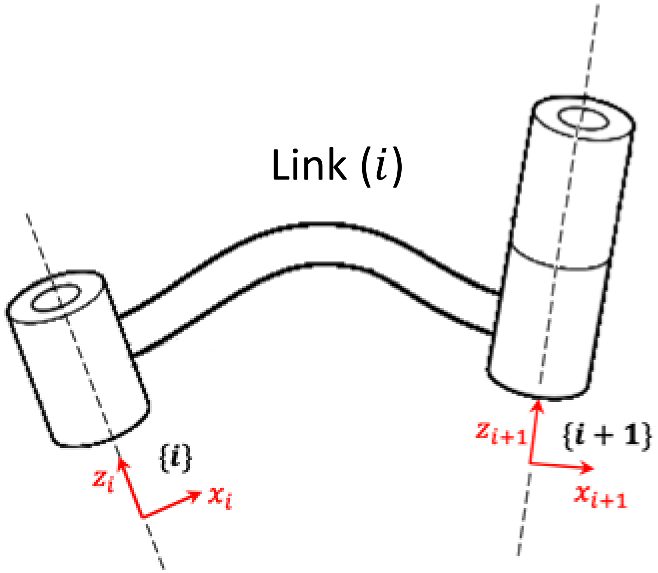
Link Velocity

Question: How to find the velocity of a certain link in a robotic arm?

- The linear velocity (v) of the origin of the frame attached to that link, and

- The rotational velocity of the link (ω)



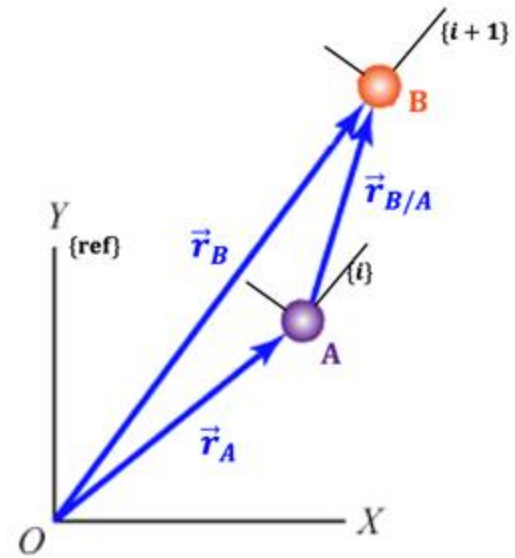
Link Velocity

Assume that you have two particles (A) and (B):

$$[{}^{\text{ref}}r_B] = [{}^{\text{ref}}r_A] + [{}^{\text{ref}}_{i}R][{}^{i}r_{B/A}]$$

Take the derivative of both sides with respect to (t)

$$[{}^{\text{ref}}v_B] = [{}^{\text{ref}}v_A] + [S({}^{\text{ref}}\omega_i)][{}^{\text{ref}}_{i}R][{}^{i}r_{B/A}] + [{}^{\text{ref}}_{i}R][{}^{i}v_{B/A}]$$



Note:

$$\frac{d[R]}{dt} = [S(\omega)][R]$$

$[S(\omega)]$ → A skew-symmetric matrix which takes the following form:

$$[S(\omega)] = \begin{bmatrix} 0 & -\omega_z & \omega_y \\ \omega_z & 0 & -\omega_x \\ -\omega_y & \omega_x & 0 \end{bmatrix}$$

Link Velocity

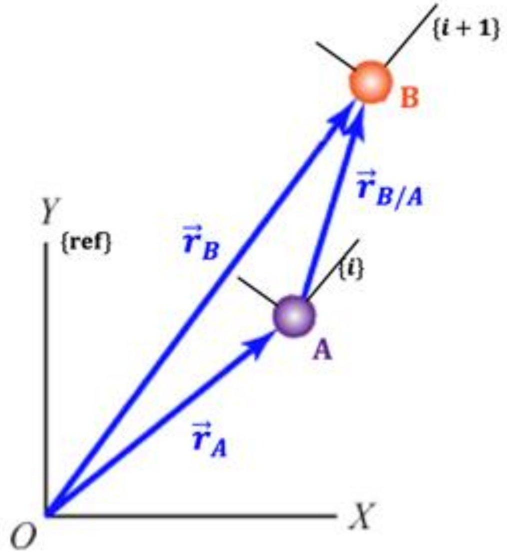
$$\left[{}^{\text{ref}}v_B\right] = \left[{}^{\text{ref}}v_A\right] + \left[S({}^{\text{ref}}\omega_i)\right]\left[{}^{\text{ref}}_{i}R\right]\left[{}^{i}r_{B/A}\right] + \left[{}^{\text{ref}}_{i}R\right]\left[{}^{i}v_{B/A}\right]$$

If both frames $\{\text{ref}\}$ and $\{i\}$ are coincident at this instant in time:

$$\left[{}^{i}v_B\right] = \left[{}^{i}v_A\right] + \left[S({}^{i}\omega_i)\right]\left[{}^{i}_{i}R\right]\left[{}^{i}r_{B/A}\right] + \left[{}^{i}_{i}R\right]\left[{}^{i}v_{B/A}\right]$$

Note: $\left[{}^{i}_{i}R\right] = [I]$

$$\left[{}^{i}v_B\right] = \left[{}^{i}v_A\right] + \left[S({}^{i}\omega_i)\right]\left[{}^{i}r_{B/A}\right] + \left[{}^{i}v_{B/A}\right]$$



Link Velocity

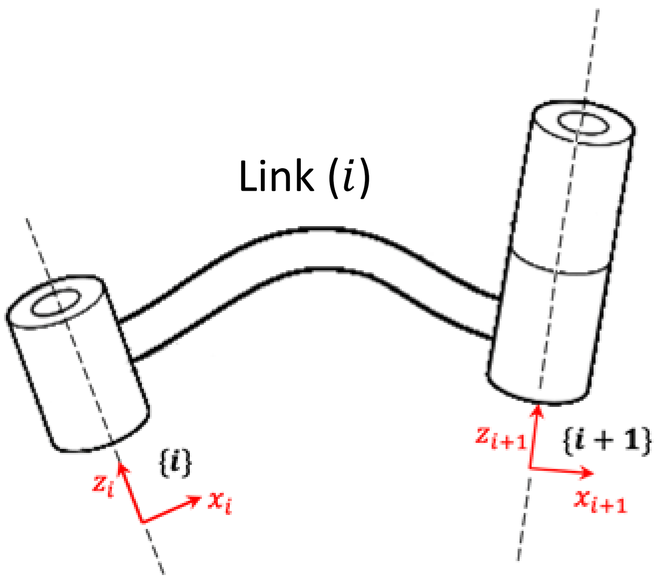
$$[\,{}^{i}v_B] = [\,{}^{i}v_A] + [\,S({}^{i}\omega_i)][{}^{i}r_{B/A}] + [{}^{i}v_{B/A}]$$

Applied to a link in the robotic arm:

Particle (A) → origin of $\{i\}$

Particle (B) → origin of $\{i+1\}$

$[{}^{i}r_{B/A}] \rightarrow [\,{}^{i}P_{i+1}\,]$



If joint $(i+1)$ is a **revolute joint**: $[\,{}^{i}v_{B/A}\,] = 0$

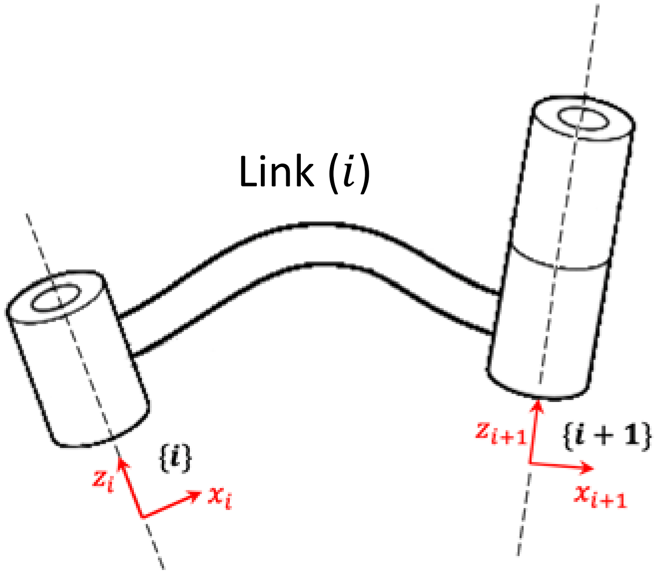
$$[\,{}^{i}v_{i+1}] = [\,{}^{i}v_i] + [\,S({}^{i}\omega_i)][{}^{i}P_{i+1}\,]$$

Link Velocity

$$[\,{}^{i}v_{i+1}] = [\,{}^{i}v_i] + [\,S({}^{i}\omega_i)][{}^{i}P_{i+1}\,]$$

Pre-multiply both sides by $[{}^{i+1}_{\;\;i}\boldsymbol{R}]$:

$$[{}^{i+1}_{\;\;i}\boldsymbol{R}][{}^{i}v_{i+1}] = [{}^{i+1}_{\;\;i}\boldsymbol{R}]([{}^{i}v_i] + [\,S({}^{i}\omega_i)][{}^{i}P_{i+1}\,])$$



$$[{}^{i+1}v_{i+1}] = [{}^{i+1}_{\;\;i}R]([{}^{i}v_i] + [\,S({}^{i}\omega_i)][{}^{i}P_{i+1}\,])$$

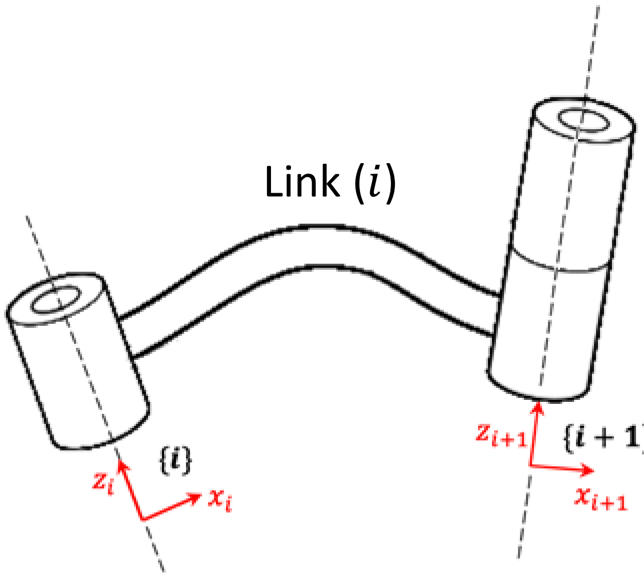
Link Velocity

$$[\,{}^{i}v_B] = [\,{}^{i}v_A] + [\,S({}^{i}\omega_i)][{}^{i}r_{B/A}] + [{}^{i}v_{B/A}]$$

Particle $(A) \rightarrow$ origin of $\{i\}$

Particle $(B) \rightarrow$ origin of $\{i+1\}$

$[{}^{i}r_{B/A}] \rightarrow [\,{}^{i}P_{i+1}\,]$

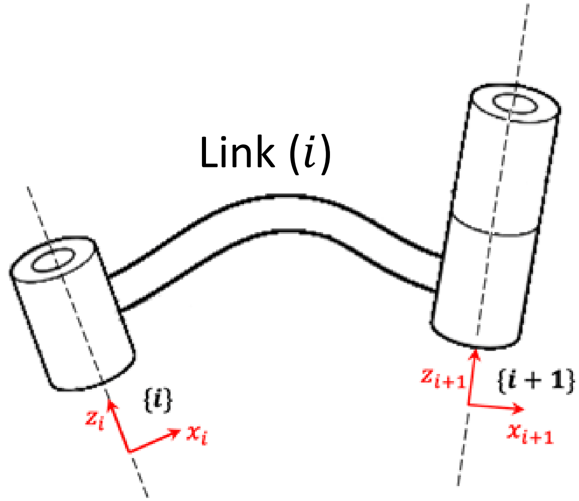


If joint $(i+1)$ is a **prismatic joint**:

$$[\,{}^{i}v_{i+1}] = [\,{}^{i}v_i] + [\,S({}^{i}\omega_i)][{}^{i}P_{i+1}\,] + [{}_{i+1}^{\;\;i}R]\,{}^{i+1}\begin{bmatrix}0\\0\\\dot{d}\end{bmatrix}$$

Link Velocity

$$[\,{}^{i}v_{i+1}] = [\,{}^{i}v_i] + [\,S({}^{i}\omega_i)][{}^{i}P_{i+1}\,] + [{}_{i+1}^{\;\;i}R]\;{}^{i+1}\begin{bmatrix}0\\0\\\dot{d}\end{bmatrix}$$



Pre-multiply both sides by $[{}_{i}^{i+1}\boldsymbol{R}]$:

$$[{}_{i}^{i+1}\boldsymbol{R}][{}^{i}v_{i+1}] = [{}_{i}^{i+1}\boldsymbol{R}]([{}^{i}v_i] + [\,S({}^{i}\omega_i)][{}^{i}P_{i+1}\,]) + [{}_{i}^{i+1}\boldsymbol{R}][{}_{i+1}^{\;\;i}R]\;{}^{i+1}\begin{bmatrix}0\\0\\\dot{d}\end{bmatrix}$$

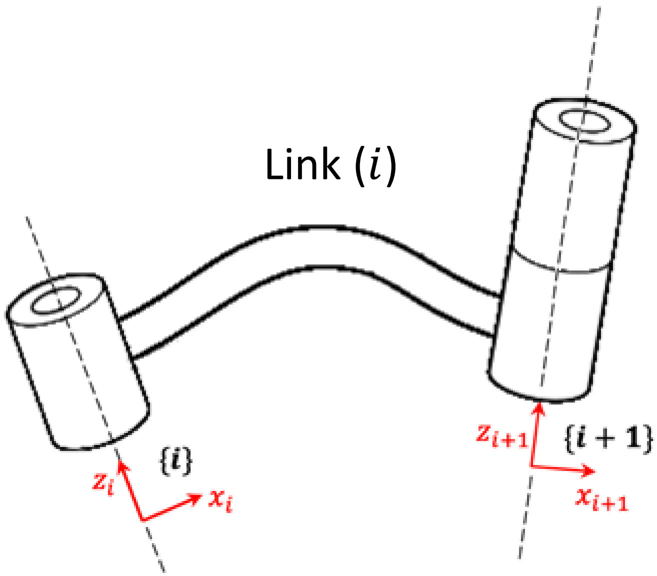
$$[{}^{i+1}v_{i+1}] = [{}_{i}^{i+1}R]([{}^{i}v_i] + [\,S({}^{i}\omega_i)][{}^{i}P_{i+1}\,]) + {}^{i+1}\begin{bmatrix}0\\0\\\dot{d}\end{bmatrix}$$

Link Velocity

Summary of linear velocity:

If joint $(i+1)$ is a revolute joint:

$$\left[{}^{i+1}v_{i+1}\right] = \left[{}^{i+1}_{\ \ i}R\right]\left(\left[{}^{i}v_i\right] + \left[\,S\left({}^{i}\omega_i\right)\right]\left[{}^{i}P_{i+1}\,\right]\right)$$

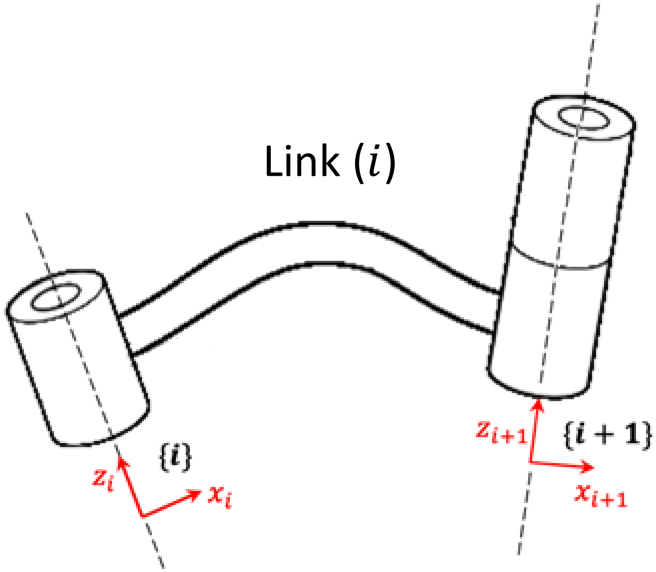


If joint $(i+1)$ is a prismatic joint:

$$\left[{}^{i+1}v_{i+1}\right] = \left[{}^{i+1}_{\ \ i}R\right]\left(\left[{}^{i}v_i\right] + \left[\,S\left({}^{i}\omega_i\right)\right]\left[{}^{i}P_{i+1}\,\right]\right) + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{d}_{i+1} \end{bmatrix}$$

Link Velocity

Angular velocity:



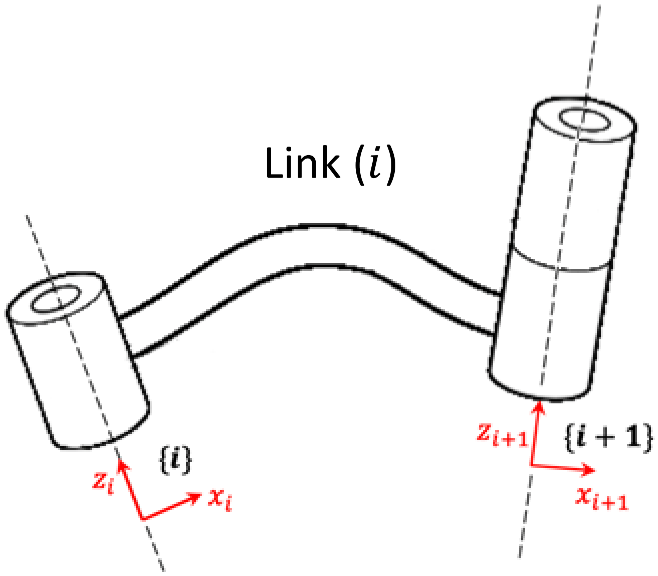
If joint $(i+1)$ is a **revolute joint**:

$$\begin{bmatrix} \text{angular} \\ \text{velocity} \\ \text{of link (i + 1)} \end{bmatrix} = \begin{bmatrix} \text{angular} \\ \text{velocity} \\ \text{of link (i)} \end{bmatrix} + \begin{bmatrix} \text{the rotational} \\ \text{velocity of} \\ \text{joint (i + 1)} \end{bmatrix}$$

$$\left[{}^{i}\omega_{i+1} \right] = \left[{}^{i}\omega_{i} \right] + \left[{}_{i+1}^{\;\;i}R \right] {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

Link Velocity

$$\left[{}^{i}\omega_{i+1}\right] = \left[{}^{i}\omega_{i}\right] + \left[{}_{i+1}^{\;i}R\right]\,{}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$



Pre-multiply both sides by $\left[{}_{\;i}^{i+1}R\right]$:

$$\left[{}_{\;i}^{i+1}R\right]\left[{}^{i}\omega_{i+1}\right] = \left[{}_{\;i}^{i+1}R\right]\left[{}^{i}\omega_{i}\right] + \left[{}_{\;i}^{i+1}R\right]\left[{}_{i+1}^{\;i}R\right]\,{}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

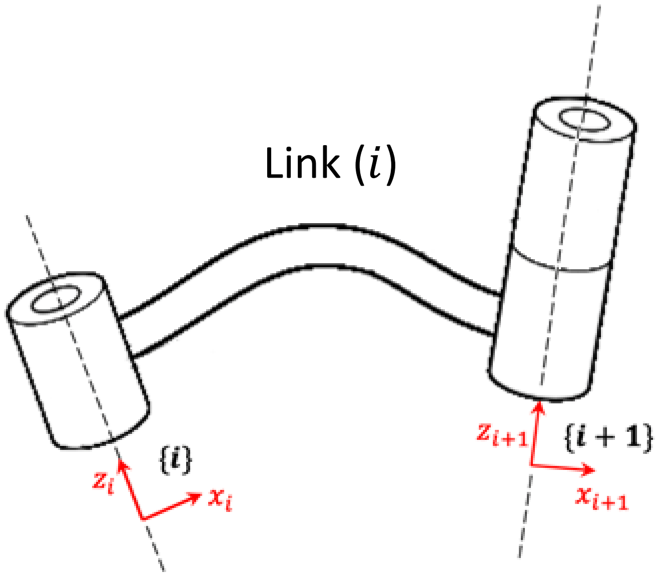
$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}_{\;i}^{i+1}R\right]\left[{}^{i}\omega_{i}\right] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

Link Velocity

If joint $(i+1)$ is a prismatic joint:

$$\begin{bmatrix}\text{angular velocity} \\ \text{of link (i + 1)}\end{bmatrix} = \begin{bmatrix}\text{angular velocity} \\ \text{of link (i)}\end{bmatrix}$$

$$[\,{}^{i}\omega_{i+1}] = [\,{}^{i}\omega_{i}]$$



Pre-multiply both sides by $[{}^{i+1}_{\;\;i}\boldsymbol{R}]$:

$$[{}^{i+1}_{\;\;i}\boldsymbol{R}][{}^{i}\omega_{i+1}] = [{}^{i+1}_{\;\;i}\boldsymbol{R}][{}^{i}\omega_{i}]$$

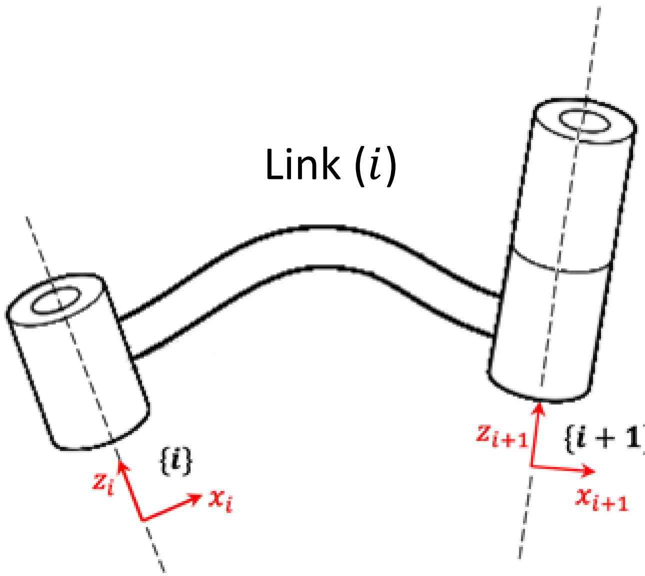
$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_{\;\;i}R][{}^{i}\omega_{i}]$$

Link Velocity

Summary of angular velocity:

If joint $(i+1)$ is a revolute joint:

$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}^{i+1}_{\ \ i}R\right]\left[{}^{i}\omega_i\right] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$



If joint $(i+1)$ is a prismatic joint:

$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}^{i+1}_{\ \ i}R\right]\left[{}^{i}\omega_i\right]$$

Link Velocity

For more details on this subject, please see:

- Introduction to Robotics: Mechanics and Control, by John J. Craig, 3rd Edition, Addison-Wesley Publishing Company, 2003.