Distance Learning Initiative

Introduction to Robotics

Workspace

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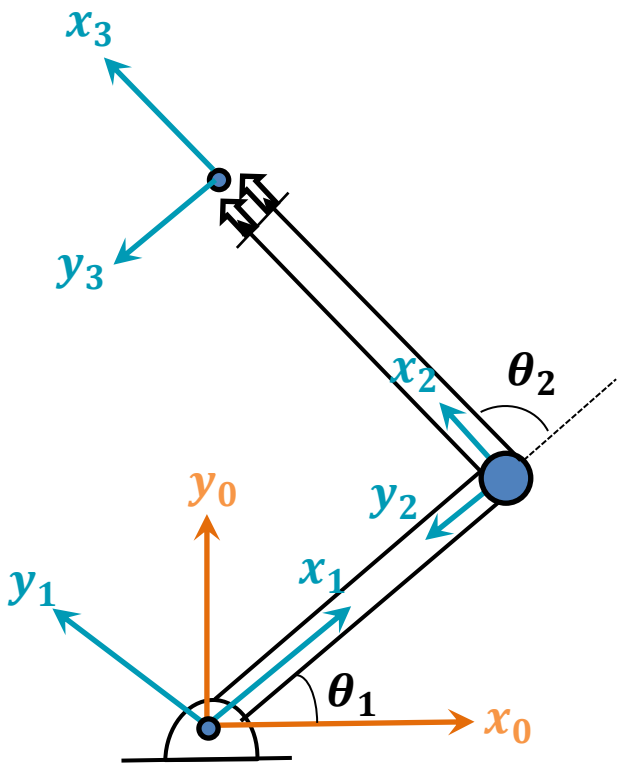
Workspace

Example:

Find the workspace for the planar 2DOF RR robotic arm shown in the figure.



Workspace

$$x^2 + y^2 = {L_1}^2 + {L_2}^2 + 2L_1L_2c_2$$

$$c_2 = \frac{x^2 + y^2 - {L_1}^2 - {L_2}^2}{2L_1L_2}$$

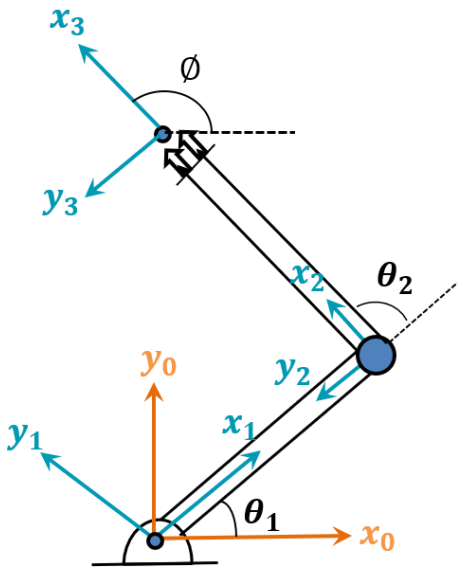
$$-1 \leq c_2 \leq 1$$

$$-1 \leq \frac{x^2 + y^2 - {L_1}^2 - {L_2}^2}{2L_1L_2} \leq 1$$

$$-2L_1L_2 \leq x^2 + y^2 - {L_1}^2 - {L_2}^2 \leq 2L_1L_2$$

$${L_1}^2 - 2L_1L_2 + {L_2}^2 \leq x^2 + y^2 \leq {L_1}^2 + 2L_1L_2 + {L_2}^2$$

$$(L_1 - L_2)^2 \leq x^2 + y^2 \leq (L_1 + L_2)^2$$



Workspace

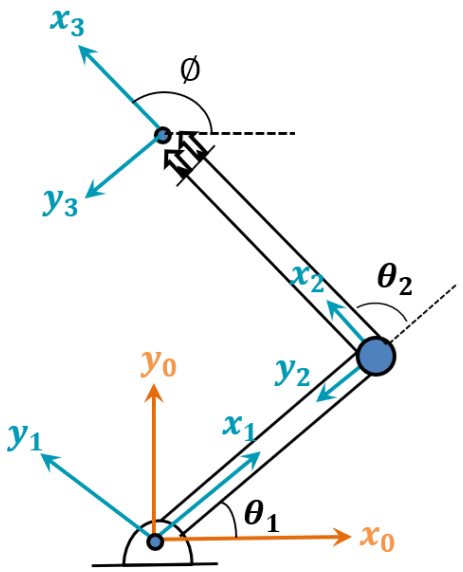
$$(L_1 - L_2)^2 \leq x^2 + y^2 \leq (L_1 + L_2)^2$$

Note that the equation of a circle is:

$$(x - x_0)^2 + (y - y_0)^2 = r^2$$

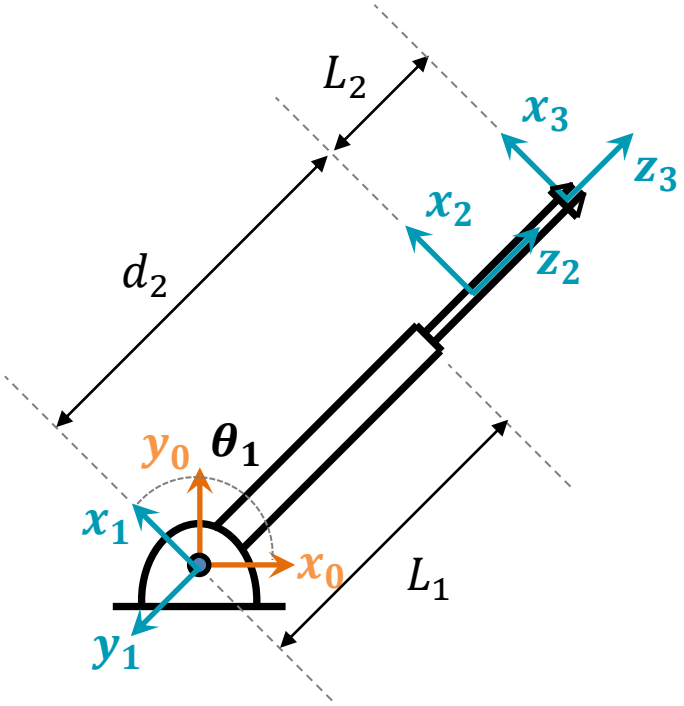
Thus the inequality above can be interpreted as:

The attainable points (x, y) by the end-effector (i.e. the workspace) create a ring (or annulus) with an inner radius of $(L_1 - L_2)$ and an outer radius of $(L_1 + L_2)$.



Example

Example: find the workspace of the planar 2 DOF RP robotic arm show in the figure?



Workspace

$$x = (L_2 + d_2)s_1 \quad (1)$$
$$y = -(L_2 + d_2)c_1 \quad (2)$$

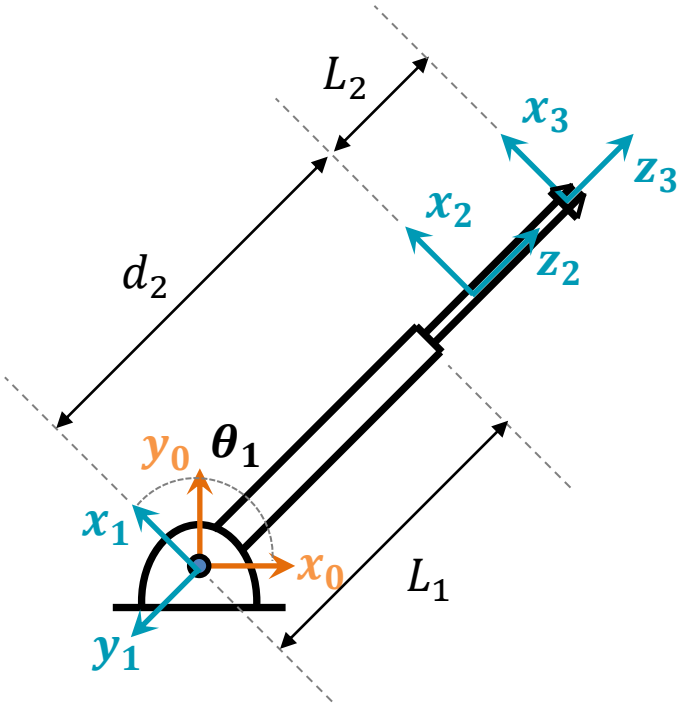
$$\theta_1 = \text{Atan2}\left(\frac{x}{-y}\right)$$

From Eqn (1):

$$x = L_2 s_1 + d_2 s_1$$

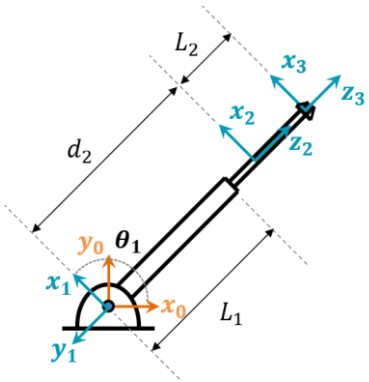
$$d_2 s_1 = x - L_2 s_1$$

$$d_2 = \frac{x - L_2 s_1}{s_1}$$



Workspace

$$x = (L_2 + d_2)s_1 \quad (1)$$
$$y = -(L_2 + d_2)c_1 \quad (2)$$



From $[\text{Eqn}(1)]^2 + [\text{Eqn}(2)]^2$:

$$x^2 + y^2 = [(L_2 + d_2)s_1]^2 + [-(L_2 + d_2)c_1]^2$$

$$x^2 + y^2 = (L_2 + d_2)^2({s_1}^2 + {c_1}^2)$$

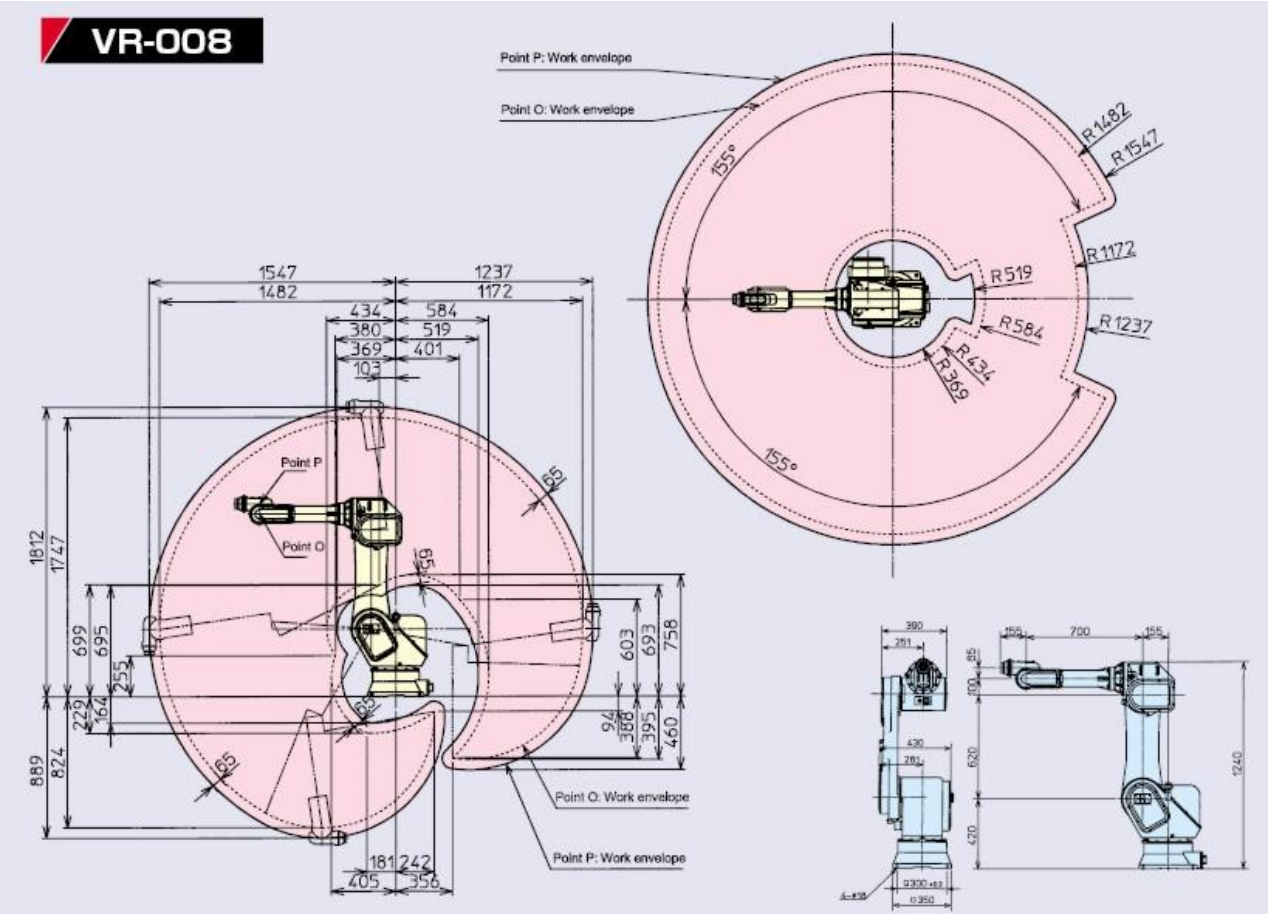
$$x^2 + y^2 = (L_2 + d_2)^2$$

This is the equation of a circle with:
center at: $(0,0)$
Radius: $r = L_2 + d_2$
Note: d_2 varies from $(d_2)_{min}$ to $(d_2)_{max}$

The workspace is a ring with:
$r_i = L_2 + (d_2)_{min}$
$r_o = L_2 + (d_2)_{max}$

Workspace

Workspace of a robotic arm is the set of all possible points that the end-effector can reach.



Panasonic VR-008GII / VR-008GLII

Source: https://robotics.ca/product/panasonic-vr-008gii-vr-008glii/

Workspace

For more details on this subject, please see:

- Introduction to Robotics: Mechanics and Control, by John J. Craig, 3rd Edition, Addison-Wesley Publishing Company, 2003.

- M. Farman, M. Al-Shaibah, Z. Aoraiath, and F. Jarrar, “Design of a three degrees of freedom robotic arm,” International Journal of Computer Applications, vol. 179(37), pp. 12-17, 2018.