Distance Learning Initiative

Introduction to Robotics

Robotic Arm Link Accelerations Example

Dr. Firas Jarrar

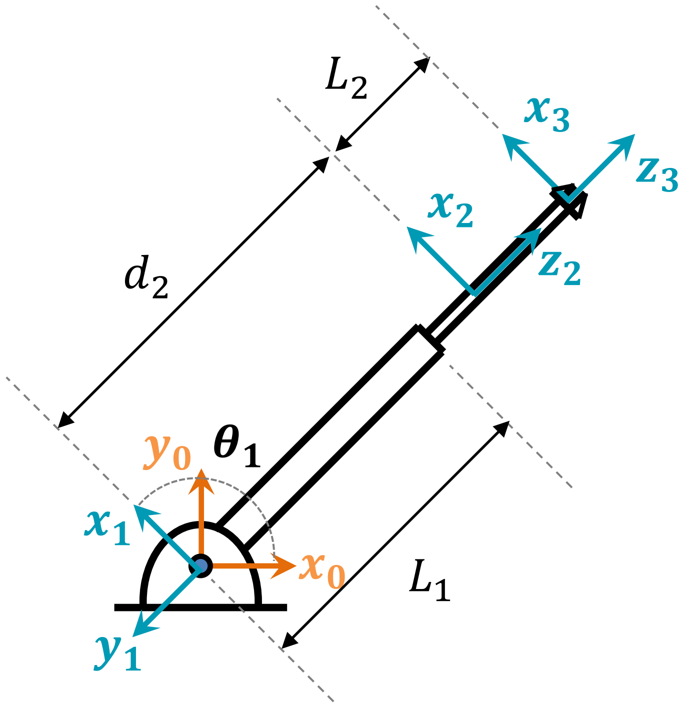
2020

Example: For the planar 2 DOF RP robotic arm, calculate the acceleration of each link and that of the end-effector as a function of the joint accelerations?

$$[{}^{0}_{1}\mathrm{T}] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{1}_{2}\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{2}_{3}\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



Link Acceleration

$$[{}^{0}_{1}T] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^{0}_{1}R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{1}_{2}T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^{1}_{2}R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[{}^{2}_{3}T] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^{2}_{3}R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

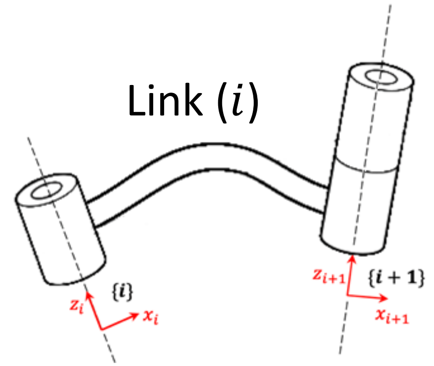
Link Acceleration

$$[{}^0_1\mathrm{T}] = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[{}^1_2\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & -d_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^1P_2] = \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix}$$

$$[{}^2_3\mathrm{T}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & L_2 \\ 0 & 0 & 0 & 1 \end{bmatrix} \rightarrow [{}^2P_3] = \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}$$

Link Acceleration



If joint $(i+1)$ is a revolute joint:

$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}^{i+1}_{\ \ i}R\right]\left[{}^{i}\omega_i\right] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

$$\left[{}^{i+1}\alpha_{i+1}\right] = \left[{}^{i+1}_{\ \ i}R\right]\left[{}^{i}\alpha_i\right] + \left[S({}^{i+1}\omega_{i+1})\right] {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix} + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_{i+1} \end{bmatrix}$$

$$\left[{}^{i+1}a_{i+1}\right] = \left[{}^{i+1}_{\ \ i}R\right]\left(\left[{}^{i}a_i\right] + \left[S({}^{i}\alpha_i)\right]\left[{}^{i}P_{i+1}\right] + \left[S({}^{i}\omega_i)\right]\left[S({}^{i}\omega_i)\right]\left[{}^{i}P_{i+1}\right]\right)$$

If joint $(i+1)$ is a prismatic joint:

$$\left[{}^{i+1}\omega_{i+1}\right] = \left[{}^{i+1}_{\ \ i}R\right]\left[{}^{i}\omega_i\right]$$

$$\left[{}^{i+1}\alpha_{i+1}\right] = \left[{}^{i+1}_{\ \ i}R\right]\left[{}^{i}\alpha_i\right]$$

$$\left[{}^{i+1}a_{i+1}\right] = \left[{}^{i+1}_{\ \ i}R\right]\left(\left[{}^{i}a_i\right] + \left[S({}^{i}\alpha_i)\right]\left[{}^{i}P_{i+1}\right] + \left[S({}^{i}\omega_i)\right]\left[S({}^{i}\omega_i)\right]\left[{}^{i}P_{i+1}\right]\right) + 2\left[{}^{i+1}_{\ \ i}R\right]\left[S({}^{i}\omega_i)\right]\left[{}^{\ \ i}_{i+1}R\right] {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{d} \end{bmatrix} + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \ddot{d} \end{bmatrix}$$

Link Acceleration

For $i = 0$:

$$[\,{}^{1}\omega_1] = [{}^{1}_{0}R][\,{}^{0}\omega_0] + \begin{bmatrix}0\\0\\\dot{\theta}_1\end{bmatrix}$$

$$[\,{}^{1}\omega_1] = \begin{bmatrix}c_1 & -s_1 & 0\\ s_1 & c_1 & 0\\ 0 & 0 & 1\end{bmatrix}^T \begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\\dot{\theta}_1\end{bmatrix}$$

$$[\,{}^{1}\omega_1] = \begin{bmatrix}0\\0\\\dot{\theta}_1\end{bmatrix}$$

$$[\,{}^{1}\alpha_1] = [{}^{1}_{0}R][\,{}^{0}\alpha_0] + [\,S({}^{1}\omega_1)]\begin{bmatrix}0\\0\\\dot{\theta}_1\end{bmatrix} + \begin{bmatrix}0\\0\\\ddot{\theta}_1\end{bmatrix}$$

$$[\,{}^{1}\alpha_1] = \begin{bmatrix}c_1 & -s_1 & 0\\ s_1 & c_1 & 0\\ 0 & 0 & 1\end{bmatrix}^T \begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}0 & -\dot{\theta}_1 & 0\\ \dot{\theta}_1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\0\\\dot{\theta}_1\end{bmatrix} + \begin{bmatrix}0\\0\\\ddot{\theta}_1\end{bmatrix}$$

$$[\,{}^{1}\alpha_1] = \begin{bmatrix}0\\0\\\ddot{\theta}_1\end{bmatrix}$$

If joint $(i+1)$ is a **revolute joint:**

$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_{\;\;i}R][{}^{i}\omega_i] + {}^{i+1}\begin{bmatrix}0\\0\\\dot{\theta}_{i+1}\end{bmatrix}$$

$$[{}^{i+1}\alpha_{i+1}] = [{}^{i+1}_{\;\;i}R][\,{}^{i}\alpha_i] + [\,S({}^{i+1}\omega_{i+1})]\;{}^{i+1}\begin{bmatrix}0\\0\\\dot{\theta}_{i+1}\end{bmatrix} + {}^{i+1}\begin{bmatrix}0\\0\\\ddot{\theta}_{i+1}\end{bmatrix}$$

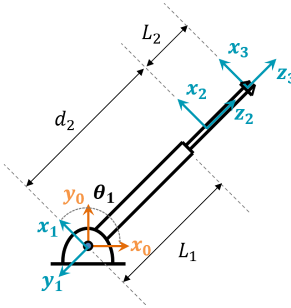
$$[{}^{i+1}a_{i+1}] = [{}^{i+1}_{\;\;i}R]\left([\,{}^{i}a_i] + [\,S({}^{i}\alpha_i)][{}^{i}P_{i+1}] + [\,S({}^{i}\omega_i)][\,S({}^{i}\omega_i)][{}^{i}P_{i+1}]\right)$$

$$[S(a)] = \begin{bmatrix}0 & -a_z & a_y\\ a_z & 0 & -a_x\\ -a_y & a_x & 0\end{bmatrix}$$

$$[{}^{0}_{1}R] = \begin{bmatrix}c_1 & -s_1 & 0\\ s_1 & c_1 & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$[{}^{1}_{2}R] = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}$$

$$[{}^{2}_{3}R] = \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$



$$[{}^{0}P_1] = \begin{bmatrix}0\\0\\0\end{bmatrix} \;;\; [{}^{1}P_2] = \begin{bmatrix}0\\-d_2\\0\end{bmatrix} \;;\; [{}^{2}P_3] = \begin{bmatrix}0\\0\\L_2\end{bmatrix}$$

Link Acceleration

$$[\,{}^{1}\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} \quad , \quad [\,{}^{1}\alpha_1] = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix}$$

Continue for $i = 0$:

$$[\,{}^{1}a_1] = [{}^{1}_{0}R]\left([\,{}^{0}a_0] + [\,S({}^{0}\alpha_0)][{}^{0}P_1] + [\,S({}^{0}\omega_0)][\,S({}^{0}\omega_0)][{}^{0}P_1]\right)$$

$$[\,{}^{1}a_1] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right)$$

$$[\,{}^{1}a_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

If joint $(i+1)$ is a **revolute joint:**

$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_{i}R][{}^{i}\omega_i] + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix}$$

$$[{}^{i+1}\alpha_{i+1}] = [{}^{i+1}_{i}R][\,{}^{i}\alpha_i] + [\,S({}^{i+1}\omega_{i+1})]\,{}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_{i+1} \end{bmatrix} + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_{i+1} \end{bmatrix}$$

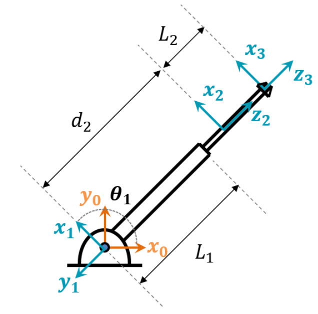
$$[{}^{i+1}a_{i+1}] = [{}^{i+1}_{i}R]\left([\,{}^{i}a_i] + [\,S({}^{i}\alpha_i)][{}^{i}P_{i+1}] + [\,S({}^{i}\omega_i)][\,S({}^{i}\omega_i)][{}^{i}P_{i+1}]\right)$$

$$[S(a)] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$[{}^{0}_{1}R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^{1}_{2}R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[{}^{2}_{3}R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[{}^{0}P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \;;\; [{}^{1}P_2] = \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix} \;;\; [{}^{2}P_3] = \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}$$

Link Acceleration

$$[\,^1\omega_1] = \begin{bmatrix}0\\0\\\dot{\theta}_1\end{bmatrix}, \; [\,^1\alpha_1] = \begin{bmatrix}0\\0\\\ddot{\theta}_1\end{bmatrix}, \; [\,^1a_1] = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

For $i = 1$:

$$[\,^2\omega_2] = [^2_1R][^1\omega_1]$$

$$[\,^2\omega_2] = \begin{bmatrix}1 & 0 & 0\\0 & 0 & -1\\0 & 1 & 0\end{bmatrix}^T \begin{bmatrix}0\\0\\\dot{\theta}_1\end{bmatrix} = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & -1 & 0\end{bmatrix}\begin{bmatrix}0\\0\\\dot{\theta}_1\end{bmatrix}$$

$$[\,^2\omega_2] = \begin{bmatrix}0\\\dot{\theta}_1\\0\end{bmatrix}$$

$$[\,^2\alpha_2] = \begin{bmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & -1 & 0\end{bmatrix}\begin{bmatrix}0\\0\\\ddot{\theta}_1\end{bmatrix}$$

$$[\,^2\alpha_2] = \begin{bmatrix}0\\\ddot{\theta}_1\\0\end{bmatrix}$$

If joint $(i+1)$ is a **prismatic joint:**

$$[^{i+1}\omega_{i+1}] = [^{i+1}_{i}R][^i\omega_i]$$

$$[^{i+1}\alpha_{i+1}] = [^{i+1}_{i}R][^i\alpha_i]$$

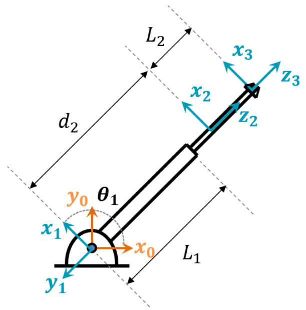
$$[^{i+1}a_{i+1}] = [^{i+1}_{i}R]([\,^ia_i] + [\,S(^i\alpha_i)][^iP_{i+1}] + [\,S(^i\omega_i)][\,S(^i\omega_i)][^iP_{i+1}]) + 2[^{i+1}_{i}R][\,S(^i\omega_i)][^{i}_{i+1}R]\;{}^{i+1}\begin{bmatrix}0\\0\\\dot{d}\end{bmatrix} + {}^{i+1}\begin{bmatrix}0\\0\\\ddot{d}\end{bmatrix}$$

$$[S(a)] = \begin{bmatrix}0 & -a_z & a_y\\a_z & 0 & -a_x\\-a_y & a_x & 0\end{bmatrix}$$

$$[^0_1R] = \begin{bmatrix}c_1 & -s_1 & 0\\s_1 & c_1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$[^1_2R] = \begin{bmatrix}1 & 0 & 0\\0 & 0 & -1\\0 & 1 & 0\end{bmatrix}$$

$$[^2_3R] = \begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$



$$[^0P_1] = \begin{bmatrix}0\\0\\0\end{bmatrix} \; ; \; [^1P_2] = \begin{bmatrix}0\\-d_2\\0\end{bmatrix} \; ; \; [^2P_3] = \begin{bmatrix}0\\0\\L_2\end{bmatrix}$$

Link Acceleration

$$[\,{}^1\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix},\ [\,{}^1\alpha_1] = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix},\ [\,{}^1a_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[\,{}^2\omega_2] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix},\ [\,{}^2\alpha_2] = \begin{bmatrix} 0 \\ \ddot{\theta}_1 \\ 0 \end{bmatrix}$$

Continue for $i = 1$:

$$[\,{}^2a_2] = [{}^2_1R]\left([\,{}^1a_1] + [\,S({}^1\alpha_1)][{}^1P_2] + [\,S({}^1\omega_1)][\,S({}^1\omega_1)][{}^1P_2]\right) + 2[{}^2_1R][\,S({}^1\omega_1)][{}^1_2R]\begin{bmatrix} 0 \\ 0 \\ \dot{d} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \ddot{d} \end{bmatrix}$$

$$[\,{}^2a_2] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}^T \left(\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -\ddot{\theta}_1 & 0 \\ \ddot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & -\dot{\theta}_1 & 0 \\ \dot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & -\dot{\theta}_1 & 0 \\ \dot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix}\right) + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}^T \begin{bmatrix} 0 & -\dot{\theta}_1 & 0 \\ \dot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ \dot{d}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \ddot{d}_2 \end{bmatrix}$$

If joint $(i+1)$ is a **prismatic joint:**

$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_{i}R][{}^{i}\omega_i]$$

$$[{}^{i+1}\alpha_{i+1}] = [{}^{i+1}_{i}R][{}^{i}\alpha_i]$$

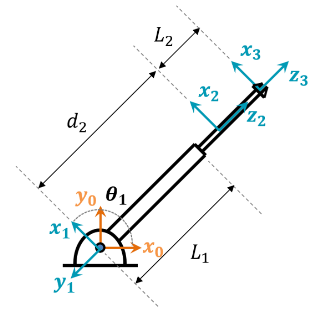
$$[{}^{i+1}a_{i+1}] = [{}^{i+1}_{i}R]\left([\,{}^ia_i] + [\,S({}^i\alpha_i)][{}^iP_{i+1}] + [\,S({}^i\omega_i)][\,S({}^i\omega_i)][{}^iP_{i+1}]\right) + 2[{}^{i+1}_{i}R][\,S({}^i\omega_i)][{}^{i}_{i+1}R]\ {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{d} \end{bmatrix} + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \ddot{d} \end{bmatrix}$$

$$[S(a)] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$[{}^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[{}^1_2R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[{}^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[{}^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}\ ;\ [{}^1P_2] = \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix}\ ;\ [{}^2P_3] = \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}$$

Link Acceleration

Continue for $i = 1$:

$$
\begin{aligned}
\left[{}^{2}a_2\right] &= \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}^T \left(\begin{bmatrix}0\\0\\0\end{bmatrix} + \begin{bmatrix}0 & -\ddot{\theta}_1 & 0\\ \ddot{\theta}_1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\-d_2\\0\end{bmatrix} + \begin{bmatrix}0 & -\dot{\theta}_1 & 0\\ \dot{\theta}_1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0 & -\dot{\theta}_1 & 0\\ \dot{\theta}_1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\-d_2\\0\end{bmatrix}\right)\\
&+ 2\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}^T \begin{bmatrix}0 & -\dot{\theta}_1 & 0\\ \dot{\theta}_1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & -1\\ 0 & 1 & 0\end{bmatrix}\begin{bmatrix}0\\0\\\dot{d}_2\end{bmatrix} + \begin{bmatrix}0\\0\\\ddot{d}_2\end{bmatrix}
\end{aligned}
$$

$$
\left[{}^{2}a_2\right] = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}\left(\begin{bmatrix}\ddot{\theta}_1 d_2\\0\\0\end{bmatrix} + \begin{bmatrix}0 & -\dot{\theta}_1 & 0\\ \dot{\theta}_1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\dot{\theta}_1 d_2\\0\\0\end{bmatrix}\right) + 2\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}\begin{bmatrix}0 & -\dot{\theta}_1 & 0\\ \dot{\theta}_1 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}0\\-\dot{d}_2\\0\end{bmatrix} + \begin{bmatrix}0\\0\\\ddot{d}_2\end{bmatrix}
$$

$$
\left[{}^{2}a_2\right] = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}\left(\begin{bmatrix}\ddot{\theta}_1 d_2\\0\\0\end{bmatrix} + \begin{bmatrix}0\\(\dot{\theta}_1)^2 d_2\\0\end{bmatrix}\right) + 2\begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}\begin{bmatrix}\dot{\theta}_1\dot{d}_2\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\\ddot{d}_2\end{bmatrix}
$$

$$
\left[{}^{2}a_2\right] = \begin{bmatrix}1 & 0 & 0\\ 0 & 0 & 1\\ 0 & -1 & 0\end{bmatrix}\begin{bmatrix}\ddot{\theta}_1 d_2\\(\dot{\theta}_1)^2 d_2\\0\end{bmatrix} + 2\begin{bmatrix}\dot{\theta}_1\dot{d}_2\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\\ddot{d}_2\end{bmatrix}
$$

$$
\left[{}^{2}a_2\right] = \begin{bmatrix}\ddot{\theta}_1 d_2\\0\\-(\dot{\theta}_1)^2 d_2\end{bmatrix} + 2\begin{bmatrix}\dot{\theta}_1\dot{d}_2\\0\\0\end{bmatrix} + \begin{bmatrix}0\\0\\\ddot{d}_2\end{bmatrix} = \begin{bmatrix}\ddot{\theta}_1 d_2 + 2\dot{\theta}_1\dot{d}_2\\0\\-(\dot{\theta}_1)^2 d_2 + \ddot{d}_2\end{bmatrix}
$$

Link Acceleration

$$[\,^1\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix} , \quad [\,^1\alpha_1] = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix} , \quad [\,^1a_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[\,^2\omega_2] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix} , \quad [\,^2\alpha_2] = \begin{bmatrix} 0 \\ \ddot{\theta}_1 \\ 0 \end{bmatrix} , \quad [\,^2a_2] = \begin{bmatrix} \ddot{\theta}_1 d_2 + 2\dot{\theta}_1 \dot{d}_2 \\ 0 \\ -(\dot{\theta}_1)^2 d_2 + \ddot{d}_2 \end{bmatrix}$$

For $i = 2$:

$$[\,^3\omega_3] = [^3_2R][^2\omega_2]$$

$$[\,^3\omega_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}$$

$$[\,^3\omega_3] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}$$

$$[\,^3\alpha_3] = [^3_2R][^2\alpha_2]$$

$$[\,^3\alpha_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \begin{bmatrix} 0 \\ \ddot{\theta}_1 \\ 0 \end{bmatrix}$$

$$[\,^3\alpha_3] = \begin{bmatrix} 0 \\ \ddot{\theta}_1 \\ 0 \end{bmatrix}$$

If joint $(i+1)$ is a **prismatic joint:**

$$[^{i+1}\omega_{i+1}] = [^{i+1}_{\ \ i}R][^i\omega_i]$$

$$[^{i+1}\alpha_{i+1}] = [^{i+1}_{\ \ i}R][^i\alpha_i]$$

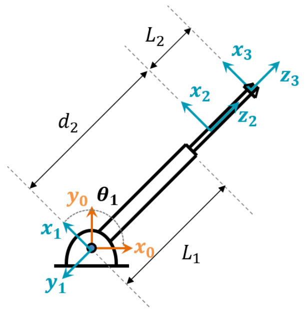
$$[^{i+1}a_{i+1}] = [^{i+1}_{\ \ i}R]([\,^ia_i] + [\,S(^i\alpha_i)][^iP_{i+1}] + [\,S(^i\omega_i)][\,S(^i\omega_i)][^iP_{i+1}]) + 2[^{i+1}_{\ \ i}R][\,S(^i\omega_i)][_{i+1}^{\ \ i}R]\,{}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \dot{d} \end{bmatrix} + {}^{i+1}\begin{bmatrix} 0 \\ 0 \\ \ddot{d} \end{bmatrix}$$

$$[S(a)] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

$$[^0_1R] = \begin{bmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[^1_2R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[^2_3R] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



$$[^0P_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \; ; \; [^1P_2] = \begin{bmatrix} 0 \\ -d_2 \\ 0 \end{bmatrix} \; ; \; [^2P_3] = \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix}$$

Link Acceleration

$$[{}^{1}\omega_1] = \begin{bmatrix}0\\0\\\dot\theta_1\end{bmatrix}, \quad [{}^{1}\alpha_1] = \begin{bmatrix}0\\0\\\ddot\theta_1\end{bmatrix}, \quad [{}^{1}a_1] = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$[{}^{2}\omega_2] = \begin{bmatrix}0\\\dot\theta_1\\0\end{bmatrix}, \quad [{}^{2}\alpha_2] = \begin{bmatrix}0\\\ddot\theta_1\\0\end{bmatrix}, \quad [{}^{2}a_2] = \begin{bmatrix}\ddot\theta_1 d_2 + 2\dot\theta_1\dot d_2\\0\\-(\dot\theta_1)^2 d_2 + \ddot d_2\end{bmatrix}$$

$$[{}^{3}\omega_3] = \begin{bmatrix}0\\\dot\theta_1\\0\end{bmatrix}, \quad [{}^{3}\alpha_3] = \begin{bmatrix}0\\\ddot\theta_1\\0\end{bmatrix}$$

Continue for $i = 2$:

$$[{}^{3}a_3] = [{}^{3}_{2}R]\left([{}^{2}a_2] + [S({}^{2}\alpha_2)][{}^{2}P_3] + [S({}^{2}\omega_2)][S({}^{2}\omega_2)][{}^{2}P_3]\right)$$

$$[{}^{3}a_3] = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}^T \left(\begin{bmatrix}\ddot\theta_1 d_2 + 2\dot\theta_1\dot d_2\\0\\-(\dot\theta_1)^2 d_2 + \ddot d_2\end{bmatrix} + \begin{bmatrix}0&0&\ddot\theta_1\\0&0&0\\-\ddot\theta_1&0&0\end{bmatrix}\begin{bmatrix}0\\0\\L_2\end{bmatrix} + \begin{bmatrix}0&0&\dot\theta_1\\0&0&0\\-\dot\theta_1&0&0\end{bmatrix}\begin{bmatrix}0&0&\dot\theta_1\\0&0&0\\-\dot\theta_1&0&0\end{bmatrix}\begin{bmatrix}0\\0\\L_2\end{bmatrix}\right)$$

If joint $(i+1)$ is a **prismatic joint:**

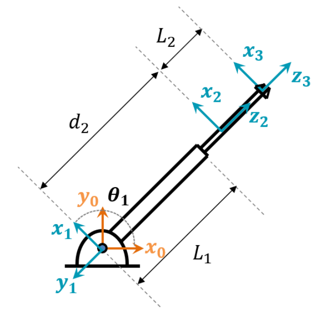
$$[{}^{i+1}\omega_{i+1}] = [{}^{i+1}_{i}R][{}^{i}\omega_i]$$

$$[{}^{i+1}\alpha_{i+1}] = [{}^{i+1}_{i}R][{}^{i}\alpha_i]$$

$$[{}^{i+1}a_{i+1}] = [{}^{i+1}_{i}R]\left([{}^{i}a_i] + [S({}^{i}\alpha_i)][{}^{i}P_{i+1}] + [S({}^{i}\omega_i)][S({}^{i}\omega_i)][{}^{i}P_{i+1}]\right) + 2[{}^{i+1}_{i}R][S({}^{i}\omega_i)][{}^{i}_{i+1}R]\,{}^{i+1}\begin{bmatrix}0\\0\\\dot d\end{bmatrix} + {}^{i+1}\begin{bmatrix}0\\0\\\ddot d\end{bmatrix}$$

$$[S(a)] = \begin{bmatrix}0&-a_z&a_y\\a_z&0&-a_x\\-a_y&a_x&0\end{bmatrix}$$

$$[{}^{0}_{1}R] = \begin{bmatrix}c_1&-s_1&0\\s_1&c_1&0\\0&0&1\end{bmatrix} \qquad [{}^{1}_{2}R] = \begin{bmatrix}1&0&0\\0&0&-1\\0&1&0\end{bmatrix} \qquad [{}^{2}_{3}R] = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$



$$[{}^{0}P_1] = \begin{bmatrix}0\\0\\0\end{bmatrix} \;;\; [{}^{1}P_2] = \begin{bmatrix}0\\-d_2\\0\end{bmatrix} \;;\; [{}^{2}P_3] = \begin{bmatrix}0\\0\\L_2\end{bmatrix}$$

Link Acceleration

Continue for $i = 2$:

$$[\,^3a_3] = [^3_2R]([\,^2a_2] + [\,S(^2\alpha_2)][^2P_3] + [\,S(^2\omega_2)][\,S(^2\omega_2)][^2P_3])$$

$$[\,^3a_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}^T \left(\begin{bmatrix} \ddot{\theta}_1 d_2 + 2\dot{\theta}_1 \dot{d}_2 \\ 0 \\ -(\dot{\theta}_1)^2 d_2 + \ddot{d}_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \ddot{\theta}_1 \\ 0 & 0 & 0 \\ -\ddot{\theta}_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \\ 0 & 0 & 0 \\ -\dot{\theta}_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \\ 0 & 0 & 0 \\ -\dot{\theta}_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ L_2 \end{bmatrix} \right)$$

$$[\,^3a_3] = \begin{bmatrix} \ddot{\theta}_1 d_2 + 2\dot{\theta}_1 \dot{d}_2 \\ 0 \\ -(\dot{\theta}_1)^2 d_2 + \ddot{d}_2 \end{bmatrix} + \begin{bmatrix} \ddot{\theta}_1 L_2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & \dot{\theta}_1 \\ 0 & 0 & 0 \\ -\dot{\theta}_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 L_2 \\ 0 \\ 0 \end{bmatrix}$$

$$[\,^3a_3] = \begin{bmatrix} \ddot{\theta}_1 d_2 + 2\dot{\theta}_1 \dot{d}_2 \\ 0 \\ -(\dot{\theta}_1)^2 d_2 + \ddot{d}_2 \end{bmatrix} + \begin{bmatrix} \ddot{\theta}_1 L_2 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -(\dot{\theta}_1)^2 L_2 \end{bmatrix}$$

$$[\,^3a_3] = \begin{bmatrix} \ddot{\theta}_1 d_2 + 2\dot{\theta}_1 \dot{d}_2 + \ddot{\theta}_1 L_2 \\ 0 \\ -(\dot{\theta}_1)^2 d_2 + \ddot{d}_2 - (\dot{\theta}_1)^2 L_2 \end{bmatrix}$$

$$[\,^3a_3] = \begin{bmatrix} \ddot{\theta}_1 (d_2 + L_2) + 2\dot{\theta}_1 \dot{d}_2 \\ 0 \\ \ddot{d}_2 - (\dot{\theta}_1)^2 (d_2 + L_2) \end{bmatrix}$$

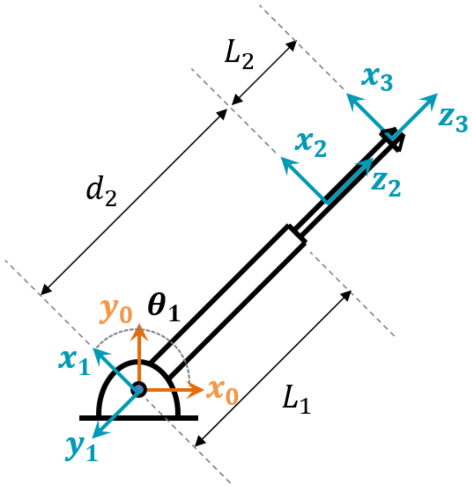
Link Acceleration

Summary:

$$[\,{}^{1}\omega_1] = \begin{bmatrix} 0 \\ 0 \\ \dot{\theta}_1 \end{bmatrix}, \quad [\,{}^{1}\alpha_1] = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix}, \quad [\,{}^{1}a_1] = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$[\,{}^{2}\omega_2] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}, \quad [\,{}^{2}\alpha_2] = \begin{bmatrix} 0 \\ \ddot{\theta}_1 \\ 0 \end{bmatrix}, \quad [\,{}^{2}a_2] = \begin{bmatrix} \ddot{\theta}_1 d_2 + 2\dot{\theta}_1 \dot{d}_2 \\ 0 \\ \ddot{d}_2 - (\dot{\theta}_1)^2 d_2 \end{bmatrix}$$

$$[\,{}^{3}\omega_3] = \begin{bmatrix} 0 \\ \dot{\theta}_1 \\ 0 \end{bmatrix}, \quad [\,{}^{3}\alpha_3] = \begin{bmatrix} 0 \\ \ddot{\theta}_1 \\ 0 \end{bmatrix}, \quad [\,{}^{3}a_3] = \begin{bmatrix} \ddot{\theta}_1 (d_2 + L_2) + 2\dot{\theta}_1 \dot{d}_2 \\ 0 \\ \ddot{d}_2 - (\dot{\theta}_1)^2 (d_2 + L_2) \end{bmatrix}$$



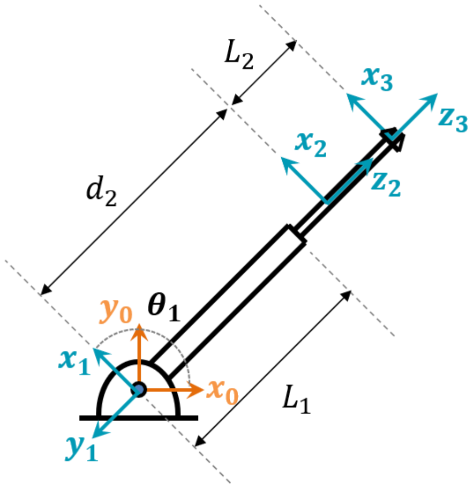
Link Acceleration

$$[\,{}^{3}\alpha_3] = \begin{bmatrix} 0 \\ \ddot{\theta}_1 \\ 0 \end{bmatrix}$$

$$[{}^{0}_{3}R] = \begin{bmatrix} c_1 & 0 & s_1 \\ s_1 & 0 & -c_1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[\,{}^{0}\alpha_3] = \begin{bmatrix} c_1 & 0 & s_1 \\ s_1 & 0 & -c_1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ \ddot{\theta}_1 \\ 0 \end{bmatrix}$$

$$\boxed{[\,{}^{0}\alpha_3] = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix}}$$



Link Acceleration

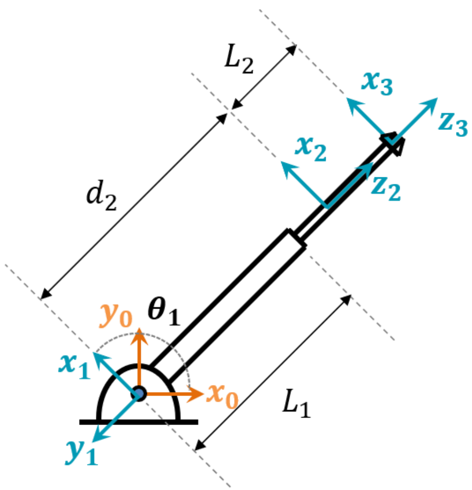
$$[\,{}^{3}a_3] = \begin{bmatrix} \ddot{\theta}_1(d_2+L_2) + 2\dot{\theta}_1\dot{d}_2 \\ 0 \\ \ddot{d}_2 - (\dot{\theta}_1)^2(d_2+L_2) \end{bmatrix}$$

$$[{}^{0}_{3}R] = \begin{bmatrix} c_1 & 0 & s_1 \\ s_1 & 0 & -c_1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$[\,{}^{0}a_3] = \begin{bmatrix} c_1 & 0 & s_1 \\ s_1 & 0 & -c_1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} \ddot{\theta}_1(d_2+L_2) + 2\dot{\theta}_1\dot{d}_2 \\ 0 \\ \ddot{d}_2 - (\dot{\theta}_1)^2(d_2+L_2) \end{bmatrix}$$

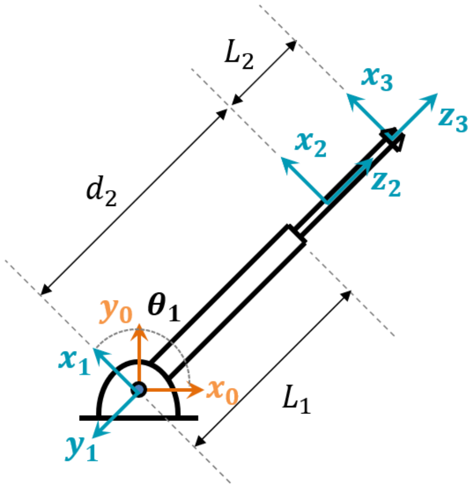
$$[\,{}^{0}a_3] = \begin{bmatrix} c_1\ddot{\theta}_1(d_2+L_2) + 2c_1\dot{\theta}_1\dot{d}_2 + s_1\ddot{d}_2 - (\dot{\theta}_1)^2(d_2+L_2)s_1 \\ s_1\ddot{\theta}_1(d_2+L_2) + 2s_1\dot{\theta}_1\dot{d}_2 - c_1\ddot{d}_2 + (\dot{\theta}_1)^2(d_2+L_2)c_1 \\ 0 \end{bmatrix}$$

$$[\,{}^{0}a_3] = \begin{bmatrix} (d_2+L_2)c_1 \\ (d_2+L_2)s_1 \\ 0 \end{bmatrix}[\ddot{\theta}_1] + \begin{bmatrix} -(d_2+L_2)s_1 \\ (d_2+L_2)c_1 \\ 0 \end{bmatrix}\left[(\dot{\theta}_1)^2\right] + \begin{bmatrix} 2c_1 \\ 2s_1 \\ 0 \end{bmatrix}[\dot{\theta}_1\dot{d}_2] + \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix}[\ddot{d}_2]$$



Link Acceleration

Summary:



$$[\,{}^{0}\alpha_3] = \begin{bmatrix} 0 \\ 0 \\ \ddot{\theta}_1 \end{bmatrix}$$

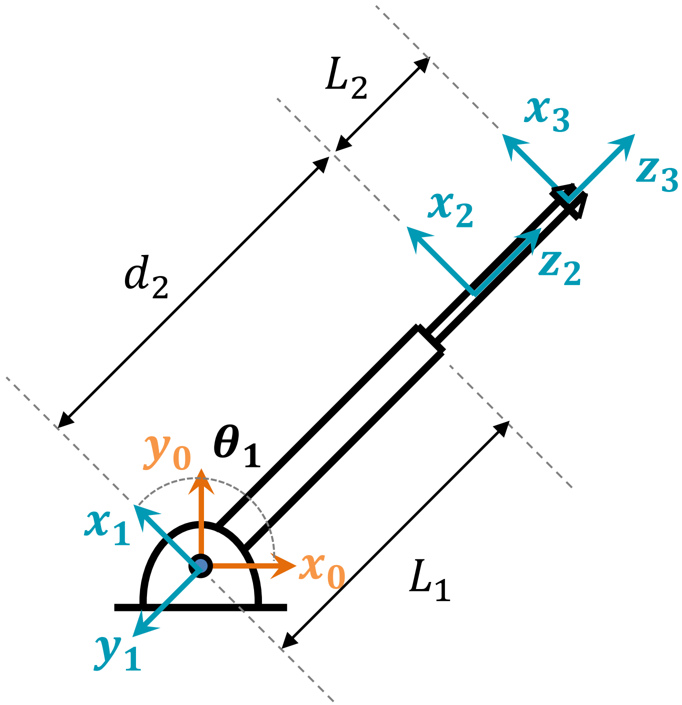
$$[\,{}^{0}a_3] = \begin{bmatrix} (d_2 + L_2)c_1 \\ (d_2 + L_2)s_1 \\ 0 \end{bmatrix} [\ddot{\theta}_1] + \begin{bmatrix} -(d_2 + L_2)s_1 \\ (d_2 + L_2)c_1 \\ 0 \end{bmatrix} \left[(\dot{\theta}_1)^2\right] + \begin{bmatrix} 2c_1 \\ 2s_1 \\ 0 \end{bmatrix} [\dot{\theta}_1 \dot{d}_2] + \begin{bmatrix} s_1 \\ -c_1 \\ 0 \end{bmatrix} [\ddot{d}_2]$$

Link Acceleration

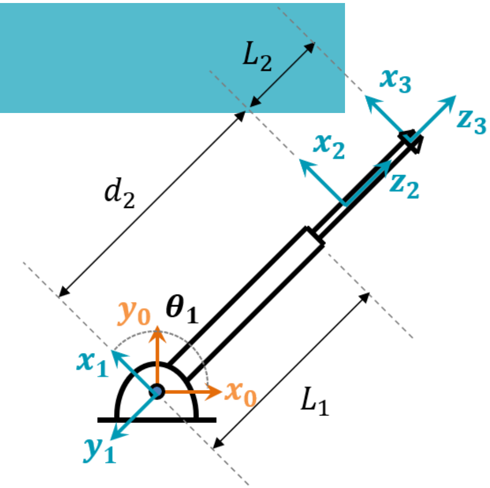
Example: For the planar 2 DOF RP robotic arm, calculate the acceleration of each link and that of the end-effector as a function of the joint accelerations? Use vector notations.

$$\vec{a}_2 = \vec{a}_1 + \vec{\alpha}_1 \times \vec{r}_{2/1} + \vec{\omega}_1 \times \left(\vec{\omega}_1 \times \vec{r}_{2/1}\right) + 2\vec{\omega}_1 \times \left(\vec{v}_{2/1}\right)_{xyz} + \left(\vec{a}_{2/1}\right)_{xyz}$$

$$\vec{a}_2 = \vec{\alpha}_1 \times \vec{r}_{2/1} - (\omega_1)^2\, \vec{r}_{2/1} + 2\vec{\omega}_1 \times \left(\vec{v}_{2/1}\right)_{xyz} + \left(\vec{a}_{2/1}\right)_{xyz}$$



Link Acceleration



$$\vec{a}_2 = \vec{\alpha}_1 \times \vec{r}_{2/1} - (\omega_1)^2\, \vec{r}_{2/1} + 2\vec{\omega}_1 \times \left(\vec{v}_{2/1}\right)_{xyz} + \left(\vec{a}_{2/1}\right)_{xyz}$$

$$\vec{a}_2$$
$$= \alpha_1 \hat{k} \times (d_2\cos(\theta_1 - 90)\hat{\imath} + d_2\sin(\theta_1 - 90)\hat{\jmath})$$
$$- (\omega_1)^2(d_2\cos(\theta_1 - 90)\hat{\imath} + d_2\sin(\theta_1 - 90)\hat{\jmath}) + 2\omega_1\hat{k}$$
$$\times \left(\dot{d}_2\cos(\theta_1 - 90)\hat{\imath} + \dot{d}_2\sin(\theta_1 - 90)\hat{\jmath}\right) + \ddot{d}_2\cos(\theta_1 - 90)\hat{\imath} + \ddot{d}_2\sin(\theta_1 - 90)\hat{\jmath}$$

$$\vec{a}_2$$
$$= \alpha_1\hat{k} \times (d_2 s_1 \hat{\imath} - d_2 c_1 \hat{\jmath}) - (\omega_1)^2(d_2 s_1 \hat{\imath} - d_2 c_1 \hat{\jmath}) + 2\omega_1\hat{k} \times \left(\dot{d}_2 s_1\hat{\imath} - \dot{d}_2 c_1 \hat{\jmath}\right) + \ddot{d}_2 s_1 \hat{\imath}$$
$$- \ddot{d}_2 c_1 \hat{\jmath}$$

$$\vec{a}_2$$
$$= \alpha_1 d_2 s_1 \hat{\jmath} + \alpha_1 d_2 c_1 \hat{\imath} - (\omega_1)^2 d_2 s_1 \hat{\imath} + (\omega_1)^2 d_2 c_1 \hat{\jmath} + 2\omega_1 \dot{d}_2 s_1 \hat{\jmath} + 2\omega_1 \dot{d}_2 c_1 \hat{\imath} + \ddot{d}_2 s_1 \hat{\imath}$$
$$- \ddot{d}_2 c_1 \hat{\jmath}$$

$$\vec{a}_2$$
$$= \left(\alpha_1 d_2 c_1 - (\omega_1)^2 d_2 s_1 + 2\omega_1 \dot{d}_2 c_1 + \ddot{d}_2 s_1\right)\hat{\imath}$$
$$+ \left(\alpha_1 d_2 s_1 + (\omega_1)^2 d_2 c_1 + 2\omega_1 \dot{d}_2 s_1 - \ddot{d}_2 c_1\right)\hat{\jmath}$$

$$\cos(\theta_1 - 90) = c_1 c_{90} + s_1 s_{90}$$
$$\cos(\theta_1 - 90) = s_1$$
$$\sin(\theta_1 - 90) = s_1 c_{90} - c_1 s_{90}$$
$$\sin(\theta_1 - 90) = -c_1$$

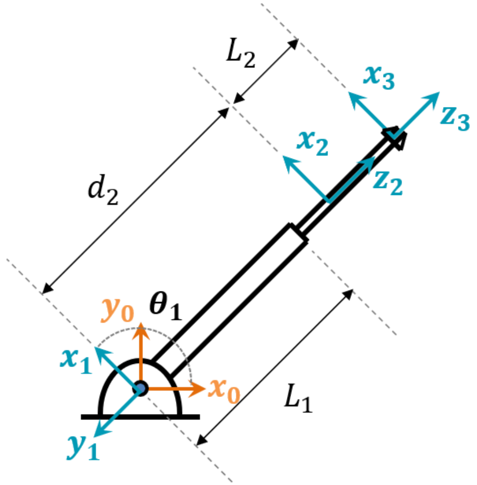
Link Acceleration

$$\vec{a}_2 = \left(\alpha_1 d_2 c_1 - (\omega_1)^2 d_2 s_1 + 2\omega_1 \dot{d}_2 c_1 + \ddot{d}_2 s_1\right)\hat{\imath} + \left(\alpha_1 d_2 s_1 + (\omega_1)^2 d_2 c_1 + 2\omega_1 \dot{d}_2 s_1 - \ddot{d}_2 c_1\right)\hat{\jmath}$$

$$\vec{a}_3 = \vec{a}_2 + \vec{\alpha}_2 \times \vec{r}_{3/2} - (\omega_2)^2\, \vec{r}_{3/2}$$

$$\vec{a}_3 = \left(\alpha_1 d_2 c_1 - (\omega_1)^2 d_2 s_1 + 2\omega_1 \dot{d}_2 c_1 + \ddot{d}_2 s_1\right)\hat{\imath} + \left(\alpha_1 d_2 s_1 + (\omega_1)^2 d_2 c_1 + 2\omega_1 \dot{d}_2 s_1 - \ddot{d}_2 c_1\right)\hat{\jmath} + \alpha_2 \hat{k} \times \left(L_2 \cos(\theta_1 - 90)\hat{\imath} + L_2 \sin(\theta_1 - 90)\hat{\jmath}\right) - (\omega_2)^2 \left(L_2 \cos(\theta_1 - 90)\hat{\imath} + L_2 \sin(\theta_1 - 90)\hat{\jmath}\right)$$

$$\vec{a}_3 = \left(\alpha_1 d_2 c_1 - (\omega_1)^2 d_2 s_1 + 2\omega_1 \dot{d}_2 c_1 + \ddot{d}_2 s_1\right)\hat{\imath} + \left(\alpha_1 d_2 s_1 + (\omega_1)^2 d_2 c_1 + 2\omega_1 \dot{d}_2 s_1 - \ddot{d}_2 c_1\right)\hat{\jmath} + \alpha_2 \hat{k} \times \left(L_2 s_1 \hat{\imath} - L_2 c_1 \hat{\jmath}\right) - (\omega_2)^2 \left(L_2 s_1 \hat{\imath} - L_2 c_1 \hat{\jmath}\right)$$



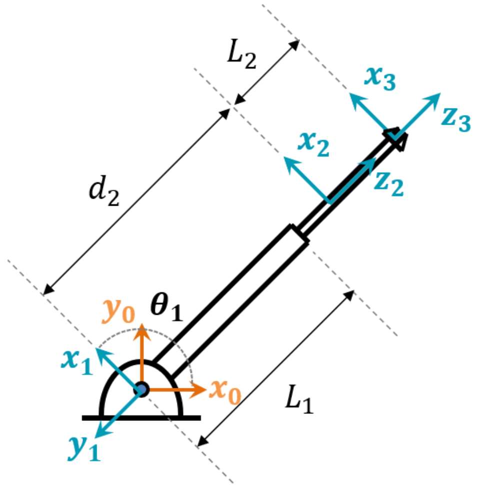
Link Acceleration

$$\vec{a}_3 = \left(\alpha_1 d_2 c_1 - (\omega_1)^2 d_2 s_1 + 2\omega_1 \dot{d}_2 c_1 + \ddot{d}_2 s_1\right)\hat{\imath} + \left(\alpha_1 d_2 c_1 - (\omega_1)^2 d_2 s_1 + 2\omega_1 \dot{d}_2 s_1 - \ddot{d}_2 c_1\right)\hat{\jmath} + \alpha_2 L_2 s_1 \hat{\jmath} + \alpha_2 L_2 c_1 \hat{\imath} - (\omega_2)^2 L_2 s_1 \hat{\imath} + (\omega_2)^2 L_2 c_1 \hat{\jmath}$$

$$\vec{a}_3 = \left(\alpha_1 d_2 c_1 - (\omega_1)^2 d_2 s_1 + 2\omega_1 \dot{d}_2 c_1 + \ddot{d}_2 s_1 + \alpha_2 L_2 c_1 - (\omega_2)^2 L_2 s_1\right)\hat{\imath} + \left(\alpha_1 d_2 c_1 - (\omega_1)^2 d_2 s_1 + 2\omega_1 \dot{d}_2 s_1 - \ddot{d}_2 c_1 + \alpha_2 L_2 s_1 + (\omega_2)^2 L_2 c_1\right)\hat{\jmath}$$

Note: $\alpha_2 = \alpha_1$ and $\omega_2 = \omega_1$

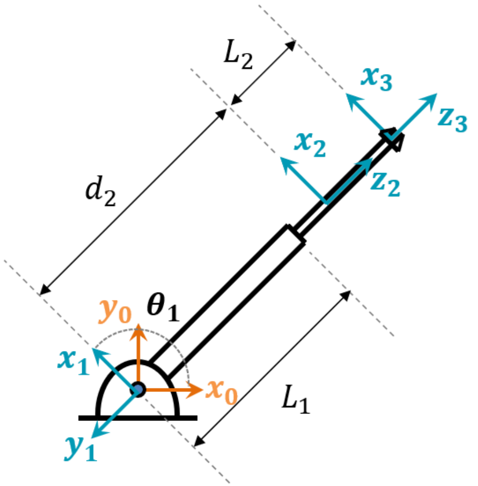
$$\vec{a}_3 = \left((d_2 + L_2) c_1 \alpha_1 - (\omega_1)^2 (d_2 + L_2) s_1 + 2\omega_1 \dot{d}_2 c_1 + \ddot{d}_2 s_1\right)\hat{\imath} + \left((d_2 + L_2) s_1 \alpha_1 + (\omega_1)^2 (d_2 + L_2) c_1 + 2\omega_1 \dot{d}_2 s_1 - \ddot{d}_2 c_1\right)\hat{\jmath}$$



Link Acceleration

$$\vec{a}_3 = \Big((d_2+L_2)c_1\alpha_1 - (\omega_1)^2(d_2+L_2)s_1 + 2\omega_1\dot{d}_2c_1 + \ddot{d}_2s_1\Big)\hat{\imath} + \Big((d_2+L_2)s_1\alpha_1 + (\omega_1)^2(d_2+L_2)c_1 + 2\omega_1\dot{d}_2s_1 - \ddot{d}_2c_1\Big)\hat{\jmath}$$

Substitute for : $\omega_1 = \dot{\theta}_1$, $\alpha_1 = \ddot{\theta}_1$

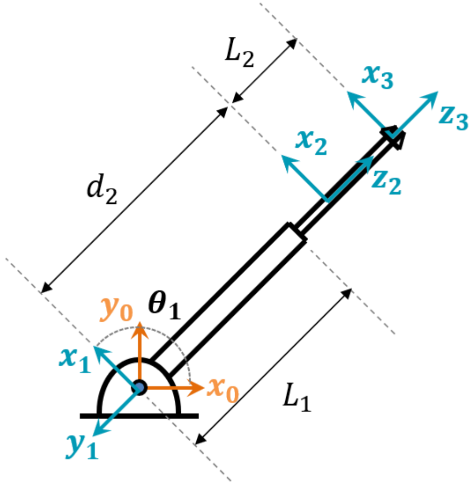


$$(a_3)_x\hat{\imath} + (a_3)_y\hat{\jmath} = \Big((d_2+L_2)c_1\ddot{\theta}_1 - \left(\dot{\theta}_1\right)^2(d_2+L_2)s_1 + 2\dot{\theta}_1\dot{d}_2c_1 + \ddot{d}_2s_1\Big)\hat{\imath} + \Big((d_2+L_2)s_1\ddot{\theta}_1 + \left(\dot{\theta}_1\right)^2(d_2+L_2)c_1 + 2\dot{\theta}_1\dot{d}_2s_1 - \ddot{d}_2c_1\Big)\hat{\jmath}$$

$$(a_3)_x = (d_2+L_2)c_1\ddot{\theta}_1 - \left(\dot{\theta}_1\right)^2(d_2+L_2)s_1 + 2\dot{\theta}_1\dot{d}_2c_1 + \ddot{d}_2s_1$$

$$(a_3)_y = (d_2+L_2)s_1\ddot{\theta}_1 + \left(\dot{\theta}_1\right)^2(d_2+L_2)c_1 + 2\dot{\theta}_1\dot{d}_2s_1 - \ddot{d}_2c_1$$

Link Acceleration



$$(a_3)_x = (d_2 + L_2)c_1\ddot{\theta}_1 - \left(\dot{\theta}_1\right)^2(d_2 + L_2)s_1 + 2\dot{\theta}_1\dot{d}_2c_1 + \ddot{d}_2s_1$$

$$(a_3)_y = (d_2 + L_2)s_1\ddot{\theta}_1 + \left(\dot{\theta}_1\right)^2(d_2 + L_2)c_1 + 2\dot{\theta}_1\dot{d}_2s_1 - \ddot{d}_2c_1$$

In matrix form:

$$\begin{bmatrix}(a_3)_x \\ (a_3)_y\end{bmatrix} = \begin{bmatrix}(d_2 + L_2)c_1\ddot{\theta}_1 - \left(\dot{\theta}_1\right)^2(d_2 + L_2)s_1 + 2\dot{\theta}_1\dot{d}_2c_1 + \ddot{d}_2s_1 \\ (d_2 + L_2)s_1\ddot{\theta}_1 + \left(\dot{\theta}_1\right)^2(d_2 + L_2)c_1 + 2\dot{\theta}_1\dot{d}_2s_1 - \ddot{d}_2c_1\end{bmatrix}$$

$$\begin{bmatrix}(a_3)_x \\ (a_3)_y\end{bmatrix} = \begin{bmatrix}(d_2 + L_2)c_1 \\ (d_2 + L_2)s_1\end{bmatrix}\left[\ddot{\theta}_1\right] + \begin{bmatrix}-(d_2 + L_2)s_1 \\ (d_2 + L_2)c_1\end{bmatrix}\left[\left(\dot{\theta}_1\right)^2\right] + \begin{bmatrix}2c_1 \\ 2s_1\end{bmatrix}\left[\dot{\theta}_1\dot{d}_2\right] + \begin{bmatrix}s_1 \\ -c_1\end{bmatrix}\left[\ddot{d}_2\right]$$

Link Acceleration

For more details on this subject, please see:

- Introduction to Robotics: Mechanics and Control, by John J. Craig, 3rd Edition, Addison-Wesley Publishing Company, 2003.